Optimal Interplant Water Networks for Industrial Zones: Addressing Interconnectivity Options Through Pipeline Merging

Sabla Y. Alnouri
Dept. of Chemical Engineering, Texas A&M University at Qatar, P.O Box 23874, Education City, Doha, Qatar
The Artie McFerrin Dept. of Chemical Engineering, Texas A&M University, College Station, TX

Patrick Linke
Dept. of Chemical Engineering, Texas A&M University at Qatar, P.O Box 23874, Education City, Doha, Qatar

Mahmoud El-Halwagi
The Artie McFerrin Dept. of Chemical Engineering, Texas A&M University, College Station, TX

DOI 10.1002/aic.14516
Published online June 11, 2014 in Wiley Online Library (wileyonlinelibrary.com)

To date, alternative design options that exist for interconnecting transmission and distribution networks have not been considered in water reuse network synthesis. Existing approaches that do incorporate piping expenses in the design of interplant water networks assign a separate pipeline for every water allocation. However, merging together common pipeline regions for the transmission of water from, or to nearby but different processing facility destinations may improve the overall water network performance not only in terms of cost efficiency but also in terms of complexity. A novel approach that is capable of accounting for pipeline merging scenarios that could exist within a water reuse network is introduced in this article. Two different pipeline branching possibilities have been introduced in this work, for the purpose of merging: (1) forward branching and (2) backward branching. The approach is implemented for the design of interplant water networks considering direct water reuse amongst several coexisting processing facilities within an industrial zone. A case study is presented to illustrate the application of the approach and its benefits. © 2014 American Institute of Chemical Engineers *AIChE J*, 60: 2853–2874, 2014

Keywords: optimization, water integration, eco-industrial parks

Introduction

Water integration methodologies offer reliable tools for identifying optimal wastewater reuse strategies that allow industries to minimize their water footprints, either individually (in-plant integration) or collectively (interplant integration). Many water integration approaches have been developed and successfully applied with a strong focus on water integration in individual plants or facilities.

Early work by Wang and Smith[1,2] led to the water-pinch analysis approach that provides insight regarding potential opportunities for wastewater minimization in process industries. Olesen and Polley[3] introduced a simple adaptation of the methodology in which additional constraints were incorporated into the water network design problem, in terms of the plant's geographical location, as well as the piping costs involved. Alva-Argáez et al.[4] developed a superstructure optimization model that includes all the possible features of a water network design, using a recursive decomposition scheme that combines insights from water-pinch analysis together with mathematical programming. Savelski and Bagajewicz[5] introduced a design methodology for water-using networks in processing plants, by investigating the necessary optimality conditions for a water allocation problem involving a single contaminant. El-Halwagi et al.[6] utilized insightful mixing and segregation principles to develop a rigorous graphical targeting approach for minimizing the overall freshwater consumption within a process by means of direct recycling schemes. Manan et al.[7] developed a water cascade analysis technique to establish the minimum water and wastewater targets for the synthesis and design of water networks. Prakash and Shenoy[8] presented an algorithm to design minimum freshwater networks for fixed flow rate problems, based on the principle of having source streams with the nearest contaminant concentrations being chosen to satisfy a particular water demand. Liu et al.[9] proposed a new method to determine the pinch points and freshwater targets for water-using networks involving a single contaminant, based on the characteristics of the pinch point in the problem, before carrying out any targeting calculations. Hu et al.[10] studied the effect of different process decomposition strategies on freshwater savings, using concentration–mass load diagrams. Lee et al.[11] explored chilled water reuse and recycle opportunities using a superstructure approach that accounts for all possible network connections, and a conflicting objective was utilized to reduce network complexity, and

Correspondence concerning this article should be addressed to P. Linke at patrick.linke@qatar.tamu.edu.

improve flexibility within the solutions obtained. Chaturvedi and Bandyopadhyay[12] proposed a multiobjective mixed integer linear programming formulation that simultaneously targets minimum freshwater utilization and maximum production in a batch process. A Pareto optimal front was used to investigate trade offs between production and fresh flows within the system.

Other contributions expanded existing water integration approaches by considering wastewater reuse amongst an existing cluster of processing facilities which is referred to as interplant water integration. Liao et al.[13] investigated the design of flexible interplant water networks by combining mathematical programming techniques with pinch analysis insights. Lovelady and El-Halwagi[14] utilized a source-interception-sink representation to develop an optimization-based approach for water allocation amongst multiple processes within a shared eco-industrial facility. Chen et al.[15] presented a novel integration scheme for interplant water integration within an industrial complex, in which both centralized and decentralized water mains were used to connect the water-using units within the individual plants. Aviso et al.[16] utilized fuzzy mathematical programming techniques to identify optimal network designs that maximize wastewater reuse amongst a cluster of plants. Chew et al.[17] introduced a new algorithm for targeting minimum fresh and waste flow rates for interplant resource conservation problems, which can also be applied for the design of water networks. Rubio-Castro et al.[18] developed a global optimal formulation for water integration in eco-industrial parks (EIP), based on a superstructure that allows the wastewater reuse within the same plant, as well as water exchange amongst different plants. Additionally, Rubio-Castro et al.[19] utilized a MINLP model to retrofit existing water networks from different plants within the same industrial zone, by accounting for both intraplant and interplant structural modifications. Boix et al.[20] formulated a Mixed-Integer Linear Programming problem based on the necessary conditions of optimality defined by Savelski and Bagajewicz,[5] for designing an EIP using three different EIP regeneration scenarios. Lee et al.[21] introduced a mathematical optimization model involving a two-stage approach, for interplant water network synthesis in which the individual processing units operate in a mix of both continuous and batch modes. More recently, Alnouri et al.[22] introduced a spatial representation for the design of interplant water networks within industrial zones, while accounting for optimum routing strategies for water allocation, by considering the layout of assigned corridor regions that available for water transport. The work was then extended to account for multiperiod planning when designing water reuse networks.[23] It was found that the design of water pipeline networks that achieve interplant integration certainly depends on the topography of an industrial zone; in terms of how the various plants and their respective processing facilities are arranged.

To date, all work has considered network connections between water sources and sinks are segregated, that is, one pipeline is associated with each connection. No work has been proposed to consider the interconnectivity options that exist for a network as a result of merging interconnecting water pipelines to reduce network complexity and capitalize on potential economies of scale. In terms of studies that involve the design of efficient pipeline networks, most contributions have been made regarding the design of gas pipelines. For instance, Wong and Larson[24] applied dynamic programming techniques to determine the optimal operating conditions for and unbranched natural gas pipeline. Graham et al.[25] performed studies on a single-phase gas network, and utilized steady-state flow and pressure distribution conditions when optimizing the design of the gas pipeline network. Flanigan[26] conducted a series of optimization problems, using the generalized reduced gradient method, for the design of optimal compressor sizes and pipeline diameters on a preselected network configuration. Baskaran and Salzborn[27] studied the problem of designing gas pipeline collection networks in a desert environment, in which no physical obstacles were considered. An efficient method for determining optimal positioning of pipeline junction points, and the respective diameter of the pipes was presented. Olorunniwo and Jensen[28,29] developed a methodology that accounts for capacity expansion in natural gas transmission networks. Almisned and Alkahtani[30] studied the design of an optimal pipeline network for transporting natural gas amongst GCC countries. Their study takes into account the type of fluid, transportation distances, location, and topography for determining all the optimization criteria required for the pipeline network. Amado[31] introduced a new modeling approach for multicommodity network flow schemes that can be utilized for sequencing refined products in pipeline systems. The overall design of the pipeline system is capable of generating the optimal sequences of batches of products and their destination, as well as the amount of product to be pumped, while satisfying the product demands. Bonnans and Spiers[32] developed a methodology for the design pipe networks via global optimization. Their study involved the investigation of a gas network optimization problem, based on the hypothesis of a stationary flow.

Enabling water reuse strategies within industrial zones requires an effective synthesis and design strategy for pipeline networks to implement interplant water transmission and distribution. Network cost is always considered a key item that would determine the viability of a developed network design. Existing water integration methods do not consider the pipeline aspect of the water network design in depth, even though a great portion of the network's total expenses would usually involve pipeline construction and maintenance costs. So far, problems involving the design of water networks associate a separate standalone pipeline with every water allocation. Such an implementation is likely not practical, especially within a typical multistakeholder setting. In a first effort toward overcoming these limitations, this article presents a novel approach to exploring interplant water integration while considering less complex interconnecting networks with merged segments. So far, all research contributions that involve interplant water network design do not incorporate such merged pipeline options as a design possibility within the network. Background and Synthesis Problem section of this article describes the synthesis problem, Methodology section outlines the methodology that has been adopted, Problem Statement and Mathematical Formulation section details the mathematical formulation, and Case Study section provides a case study illustration.

Background and Synthesis Problem

Pipelines are the prevalent infrastructures to facilitate low-cost material exchange across processing locations. The pipeline construction costs depend on the material of

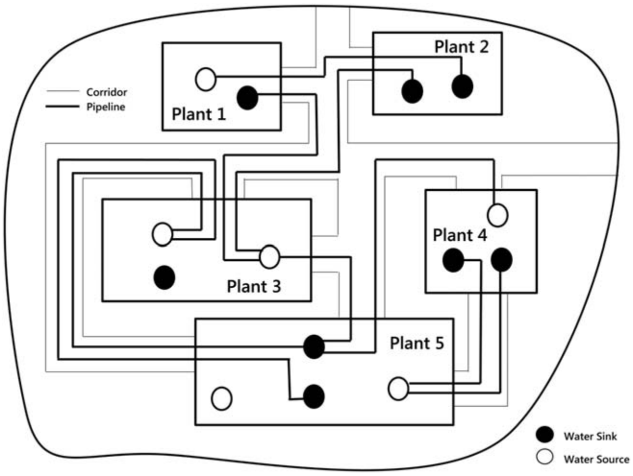

Figure 1. Typical output of a source-sink mapping activity, for the design of interplant water networks.

construction, diameter and length of the pipeline being assembled, and their implementation (surface or buried).Parallel pipelines of small diameters are typically more expensive to construct, maintain, and operate compared to large diameter pipelines conveying the same water flow. The design of effective and cost efficient pipeline networks for interplant water transmission and distribution is very important, because economics and complexity play an important role in the development of sustainable strategies for water reuse. The exploration of pipeline design alternatives within the boundaries of industrial zones is necessary to identify effective solutions from the different options that exist for assembling interconnecting networks.

Even though existing interplant water integration methods may reveal substantial water savings through wastewater reuse amongst an existing cluster of plants, water transmission via pipelines is often a major cost item. A typical output of a water integration approach considers each source-sink interconnection to constitute a separate pipeline (Figure 1). Given the spatial layout of an industrial zone with the requirements to maintain pipelines within defined corridor regions,[22] parallel pipelines can be expected to emerge when implementing optimal water reuse allocations amongst a cluster of plants. Moreover, the stakeholders responsible for the development of water networks across an industrial zone are typically different entities from the ones owning or operating the facilities within the city. Therefore, a pipeline network to be implemented in such a multistakeholder setting would require acceptably low complexity that is unlikely to be achieved if each source-sink connection would require a separate pipeline.

The complexity of a water network design could often be reduced through fewer connections, by identifying pipelines with common segments that are transporting water of similar quality to different but relatively close destinations. Moreover, substantial economies of scale are often achieved when transporting materials in bulk. These economies of scale typically result in low operating costs, when compared to the construction costs entailed. Pipelines are often attributed with the ability to effectively transport large quantities of material from one location to another, since a slight increase in the diameter of a pipeline can exponentially enhance its respective transportation capacity. This makes it more efficient to build one large pipeline rather than two or more small pipelines in many situations. Moreover, networks involving relatively larger pipelines are often easier to operate and maintain, and their governance simplifies when fewer pipes and segments are involved. Conversely, it might in some cases be more economical to build parallel piping arrangements for smaller systems that do not require high transmission capacities or where water qualities significantly differ.

The identification of low cost pipeline networks for a given industrial zone water integration challenge requires the ability to represent and assess the various possible network options. Given that existing approaches only consider water networks with segregated source-sink connecting pipelines, the purpose of this work is to develop a representation for use in water integration, that is, capable of capturing the opportunities for merging pipelines so as to enable the screening of less complex pipeline networks in the course of determining optimal water integration strategies. The efficiency of implementing merged pipeline scenarios is compared to results from previous work,[22] which assigns a separate pipeline for each water allocation.

Methodology

As aforementioned, all current approaches that involve synthesis and design of water networks associate a separate pipeline with every water allocation. We refer to an "unmerged connectivity" when we describe such networks. This section presents a methodology to enable the design of water networks while incorporating merged pipeline transmission options, amongst several coexisting processing facilities within an industrial zone. For the purpose of keeping the methodology illustration relatively simple in this article, this work considers the case of direct water reuse to achieve water integration across plants in an industrial zone. However, it should be noted that the same principles that are introduced in this article can be extended and applied for cases in which water regeneration and reuse strategies are explored for water integration.

A strategy for the systematic development of pipeline merging and assembling strategies in interplant water networks is required to capture alternative pipeline network options. We first identify the different types connectivity involved within the network for direct water reuse: (a) source-to-sink, (b) fresh-to-source, and (c) sink-to-waste. For

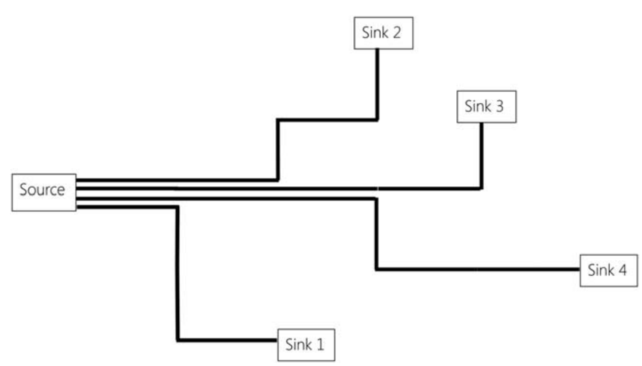

Figure 2. An unmerged pipeline connectivity demonstration for a given water source, distributing water to several nearby water sinks.

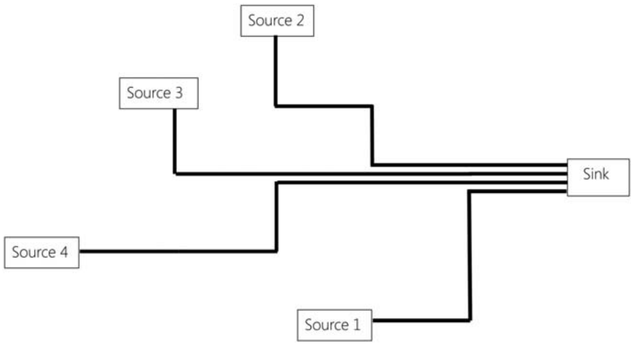

Figure 3. An unmerged pipeline connectivity demonstration for a given water sink, receiving water from several nearby water sources.

a given water source (or freshwater source) feeding into multiple sinks, a common unmerged interconnectivity scenario would usually involve separate pipelines to the individual water sinks, as illustrated in Figure 2. Similarly, for a given water sink (or wastewater discharge sink), an unmerged scenario would involve water being received from several water sources, as illustrated in Figure 3. Merging pipelines can result in various stream mixing options. Such mixing of wastewater streams with different qualities might hinder their usage in a number of potential water sinks, when mixed in the same pipeline with other water streams with different contamination levels. This can be avoided with a merging scenario that yields no change in quality, compared to the case of being transported in a separate pipe scenario; hence, the merged pipeline will be associated with a uniform water quality. This case considers only pipelines that originate from same water source location to be merged together. Similarly, pipelines to the same water sink destination can also be merged without undesired mixing. The two merging options lead to two different pipeline branching schemes that could be adopted, while avoiding any mixing in between water qualities within the pipelines.

Forward branching scheme

This scheme involves a given water source (or freshwater mains) being distributed to several nearby water sinks. Figure 4 illustrates the concept of forward branching to determine merged pipeline segments. The pipeline is constructed using relatively larger diameters at the very beginning of the transmission, and narrows down to smaller diameters to accommodate the changes in flow rates from section to section. The forward branching applies to source-to-sink and fresh-to source connectivity categories.

Backward branching scheme

This scheme involves water from several nearby water sources being collectively transmitted to a given water sink (or wastewater discharge mains). Figure 5 illustrates the concept of backward branching to determine merged pipeline segments. The pipeline is constructed using relatively smaller diameters at the beginning of the transmission, and increases to larger diameters as flows increase. Backward branching applies to source-to-sink and sink-to-waste connectivity categories.

Figures 4 and 5 show that regardless of the branching scheme that is selected for assembling a merged pipeline, both options share several common characteristics. Merged pipelines feature nodes that connect the various branches together, with each node intersection resulting in a flow and size (diameter) change. Hence, every pipeline branch is defined between two consecutive nodes, and is associated with a different size when compared to both preceding and subsequent branches. In this work, all pipeline nodes have been defined according to levels, which are named according to the degree of branching involved. For instance, first level nodes consist of the first set of nodes that form pipelines braches, and have no preceding nodes within the pipeline, except the starting point, whereas a second level node would originate from a preceding first level node and so on. Figure 6 illustrates the node level classification procedure that has been followed which defines the endpoints of the various segments or branches within a merged pipeline. All first level branches in the pipeline are formed by connecting the point(s) from which the pipeline originates to the different first level nodes that exist within the pipeline. Similarly, All second level branches in the pipeline are formed by connecting first level nodes to second level nodes that exist within the pipeline. In case further branching is considered, third level nodes would then form another set of third branches,

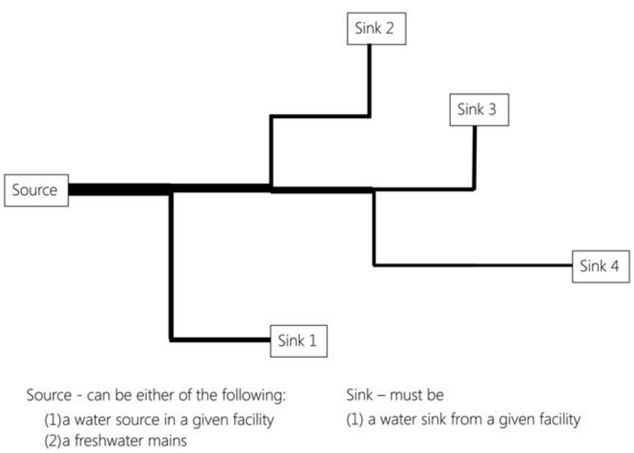

Figure 4. A merged pipeline connectivity demonstration via forward branching, for a given water source, distributing water to several nearby water sinks.

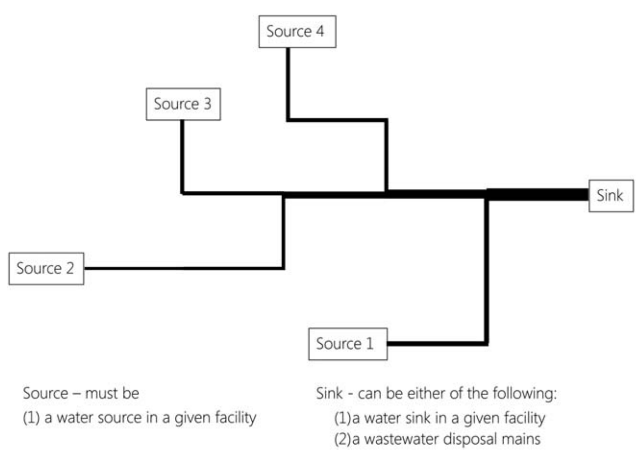

Figure 5. A merged pipeline connectivity demonstration via backward branching, for a given water sink, receiving water from several nearby water sources.

by connecting to third level nodes. The procedure is repeated until the different node levels consistently connect to successive levels, and keep forming new sets of pipeline branches, up until reaching the destination point(s).

Problem Statement and Mathematical Formulation

The problem statement can be summarized as follows. Given an industrial zone consisting of a cluster of plants P, each containing its own set of water sources SU_p and water sinks SN_p, it is required to develop a strategy for optimal water reuse across the different water processes subject to minimizing the total piping and freshwater costs of the interplant water network design. In this work, the optimal solutions are sought for a direct water reuse strategy that achieves a cost-optimal network, while taking into account the various pipeline merging scenarios that could be incorporated into the network design, for interplant water transmission and distribution. The objective function is specified as

$$\begin{aligned}
&\textit{Minimize.}\quad \gamma[\sum\sum\sum\sum_{p\in P\ \ i\in SU_p j\in SN_p a\in X_{ip,jp'}}\alpha\left(DI^{a}_{ip,jp'}\right)^{\beta}L^{a}_{ip,jp'}\\
&+\sum\sum\sum_{p\in P\ \ i\in SU_p a\in X_{ip}}\alpha\left(DI^{a}_{ip}\right)^{\beta}L^{a}_{ip}\\
&+\sum\sum\sum_{p\in P\ \ j\in SN_p a\in X_{jp}}\alpha\left(DI^{a}_{jp}\right)^{\beta}L^{a}_{jp}+\sum\sum\sum\sum_{p\in P\ \ i\in SU_p j\in SN_p a\in X_{ip,jp'}\ \ b\in Y_{ip,jp'}}\alpha\left(DI^{a,b}_{ip,jp'}\right)^{\beta}L^{a,b}_{ip,jp'}\\
&+\sum\sum\sum_{p\in P\ \ i\in SU_p a\in X_{ip}\ \ b\in Y_{ip}}\alpha\left(DI^{a,b}_{ip}\right)^{\beta}L^{a,b}_{ip}+\sum\sum\sum_{p\in P\ \ j\in SN_p a\in X_{jp}\ \ b\in Y_{jp}}\alpha\left(DI^{a,b}_{jp}\right)^{\beta}L^{a,b}_{jp}\\
&+\sum\sum\sum\sum_{p\in P\ \ i\in SU_p j\in SN_p\ \ a\in X_{ip,jp'}\ \ b\in Y_{ip,jp'}\ \ c\in Z_{ip,jp'}}\alpha\left(DI^{a,b,c}_{ip,jp'}\right)^{\beta}L^{a,b,c}_{ip,jp'}\\
&+\sum\sum\sum_{p\in P\ \ i\in SU_p a\in X_{ip}\ \ b\in Y_{ip}\ \ c\in Z_{ip}}\alpha\left(DI^{a,b,c}_{ip}\right)^{\beta}L^{a,b,c}_{ip}\\
&+\sum\sum\sum_{p\in P\ \ j\in SN_p a\in X_{jp}\ \ b\in Y_{jp}\ \ c\in Z_{jp}}\alpha\left(DI^{a,b,c}_{jp}\right)^{\beta}L^{a,b,c}_{jp}+\cdots\\
&+\sum\sum\sum\sum_{p\in P\ \ i\in SU_p j\in SN_p\ \ a\in X_{ip,jp'}\ \ b\in Y_{ip,jp'}\ \ c\in Z_{ip,jp'}\ldots(n-1)\in(N-1)_{ip,jp'}\ \ n\in N_{ip,jp'}}\alpha\left(DI^{a,b,c,..,n-1,n}_{ip,jp'}\right)^{\beta}L^{a,b,c,..,n-1,n}_{ip,jp'}\\
&+\sum\sum\sum_{p\in P\ \ i\in SU_p a\in X_{ip}\ \ b\in Y_{ip}\ \ c\in Z_{ip}\ldots(n-1)\in(N-1)_{ip}\ \ n\in N_{ip}}\alpha\left(DI^{a,b,c,..,n-1,n}_{ip}\right)^{\beta}L^{a,b,c,..,n-1,n}_{ip}\\
&+\sum\sum\sum_{p\in P\ \ j\in SN_p a\in X_{jp}\ \ b\in Y_{jp}\ \ c\in Z_{jp}\ldots(n-1)\in(N-1)_{jp}\ \ n\in N_{jp}}\alpha\left(DI^{a,b,c,..,n-1,n}_{jp}\right)^{\beta}L^{a,b,c,..,n-1,n}_{jp}]\\
&+H_yC^{\text{FRESH}}\sum\sum_{p\in P\ \ j\in SN_p}F_{jp}
\end{aligned}\qquad (1)$$

The water integration problem is subject to a number of constraints that involve total mass balances around all water sources (Eq. 2) and sinks (Eq. 3), in which the individual flow terms must equal all given water source flows (W_{ip}), and the specified sink flows (G_{jp}), respectively. Additionally, the network is also subject to component mass balances around the water sinks, as described by Eq. 4. Equation 5 sets limits on the allowable sink contaminant range, according to the maximum and minimum pollutant limits that are allowed into each sink. Additionally, Eqs. 6–8 associate all flow rate variables with a non-negativity condition. Equations 2–8 were all based on direct water reuse formulations

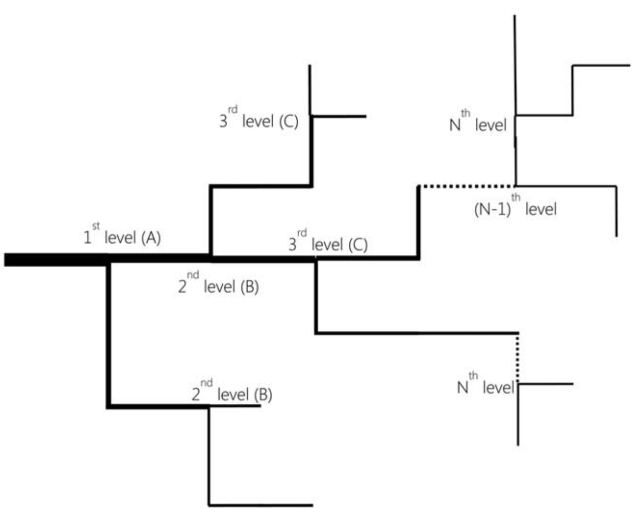

Figure 6. Node level illustration (for both forward and backward branching).

$$\sum\sum_{p\in P\ \ j\in SN_p}M_{ip,jp'}+D_{ip}=W_{ip}\quad \forall p,p'\in P\quad \forall i\in SU_p \quad (2)$$

$$\sum\sum_{p\in P\ \ i\in SU_p}M_{ip,jp'}+F_{jp}=G_{jp}\quad \forall p,p'\in P\quad \forall j\in SN_p \quad (3)$$

$$\begin{gathered}\sum\sum_{p\in P\ \ i\in SU_p}M_{ip,jp'}\ x^{\text{Source}}_{c,ip}+F_{jp}x_c{}^{\text{FRESH}}=G_{jp}z^{in}_{c,jp}\\ \forall p,p'\in P;\quad \forall j\in SN_p;\ \forall c\in C\end{gathered}\quad (4)$$

$$z^{\min}_{c,jp}\le z^{in}_{c,jp}\le z^{\max}_{c,jp} \quad (5)$$

$$M_{ip,jp'}\ge 0\ \forall p,p'\in P;\ \forall j\in SN_p;\ \forall i\in SU_p \quad (6)$$

$$D_{ip}\ge 0\quad \forall p\in P;\ \forall i\in SU_p \quad (7)$$

$$F_{jp} \geq 0 \quad \forall p \in P;\ \forall j \in \mathrm{SN}_p \tag{8}$$

Additionally, pipe diameters are calculated using Eq. 9, according to the recommended velocity ranges by Peters et al.,[33] using the mass flow rate (kg/s) of each respective stream. All diameters were then rounded up to the nearest size, so as to reflect the use of a standardized, instead of customized pipe sizes

$$\mathrm{DI} = \mathrm{Roundup}\left[0.363\left(\left(\frac{\mathrm{Flow}}{\rho}\right)^{0.45} \rho^{0.13}\right)\right] \tag{9}$$

In addition to the above source-sink mapping formulation for direct water reuse, the constraints relating to pipeline merged segments are derived below. Each merging scenario can is implemented separately.

Forward branching formulation

Equations 10–35 below detail the mathematical formulation associated with a forward branching scheme in a pipeline.

The flow allocated from source i in plant p to sink j in plant p' $(M_{ip,jp'})$ must equal the summation of all flows $(M^a_{ip,jp'})$ from the various branches that connect source i in plant p to all 1st level nodes a, associated with the stream connection

$$\sum_{a=1}^{X_{ip,jp'}} M^a_{ip,jp'} = M_{ip,jp'} \quad \forall i \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p;\ \forall p, p' \in P \tag{10}$$

The flow allocated from the freshwater mains to sink j in plant p' (F_{jp}) must equal the summation of all flows $\left(F^a_{jp}\right)$ from the various branches that connect the fresh mains to all 1st level nodes a, associated with the stream connection

$$\sum_{a=1}^{X_{jp}} F^a_{jp} = F_{jp} \quad \forall j \in \mathrm{SN}_p;\ \forall p \in P \tag{11}$$

The flows allocated from each of the 1st level nodes in the stream that connects source i in plant p to sink j in plant p' $(M^a_{ip,jp'})$ must equal the summation of all flows $(M^a_{ip,jp'})$ from the various branches that connect each 1st level node a, to all 2nd level nodes b associated with the stream connection

$$\sum_{b=1}^{Y_{ip,jp'}} M^{a,b}_{ip,jp'} = M^a_{ip,jp'} \quad \forall a \in X_{ip,jp'}; \forall i \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p;\ \forall p, p' \in P \tag{12}$$

The flows allocated from each of the 1st level nodes in the stream that connects the freshwater mains to sink j in plant p' (F^a_{jp}) must equal the summation of all flows $(F^{a,b}_{jp})$ from the various branches that connect each 1st level node a, to all 2nd level nodes b associated with the stream connection

$$\sum_{b=1}^{Y_{jp}} F^{a,b}_{jp} = F^a_{jp} \quad \forall a \in X_{jp};\ \forall j \in \mathrm{SN}_p;\ \forall p \in P \tag{13}$$

The flows allocated from each of the 2nd level nodes in the stream that connects source i in plant p to sink j in plant p' $(M^{a,b}_{ip,jp'})$ must equal the summation of all flows $(M^{a,b,c}_{ip,jp'})$ from the various branches that connect each 2nd level node b, to all 3rd level nodes c associated with the stream connection

$$\sum_{c=1}^{Z_{ip,jp'}} M^{a,b,c}_{ip,jp'} = M^{a,b}_{ip,jp'} \quad \forall a \in X_{ip,jp'};\ \forall b \in Y_{ip,jp'}; \quad \forall i \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p;\ \forall p, p' \in P \tag{14}$$

The flows allocated from each of the 2nd level nodes in the stream that connects the freshwater mains to sink j in plant p' $(F^{a,b}_{jp})$ must equal the summation of all flows $(F^{a,b,c}_{jp})$ from the various branches that connect each 2nd level node b, to all 3rd level nodes c associated with the stream connection

$$\sum_{c=1}^{Z_{jp}} F^{a,b,c}_{jp} = F^{a,b}_{jp} \quad \forall a \in X_{jp};\ \forall b \in Y_{jp}; \forall j \in \mathrm{SN}_p;\ \forall p \in P \tag{15}$$

The flows allocated from each of the $(n-1)$th level nodes in the stream that connects source i in plant p to sink j in plant p' $(M^{a,b,c,..,n-1}_{ip,jp'})$ must equal the summation of all flows $(M^{a,b,c,..,n}_{ip,jp'})$ from the various branches that connect each $(n-1)$th level node, to all nth level nodes associated with the stream connection

$$\sum_{n=1}^{N_{ip,jp'}} M^{a,b,c,..,n-1,n}_{ip,jp'} = M^{a,b,c,..,n-1}_{ip,jp'} \quad \forall a \in X_{ip,jp'};\ \forall b \in Y_{ip,jp'}; \quad \forall c \in Z_{ip,jp'}\ ..\forall (n-1) \in (N-1)_{ip,jp'};\ \forall i \in \mathrm{SU}_p; \quad \forall j \in \mathrm{SN}_p;\ \forall p, p' \in P \tag{16}$$

The flows allocated from each of the $(n-1)$th level nodes in the stream that connects the freshwater mains to sink j in plant $p'(F^{a,b,c,..,n-1}_{jp})$ must equal the summation of all flows $(F^{a,b,c,..,n-1,n}_{jp})$ from the various branches that connect each $(n-1)$th level node, to all nth level nodes associated with the stream connection

$$\sum_{n=1}^{N_{jp}} F^{a,b,c,..,n-1,n}_{jp} = F^{a,b,c,..,n-1}_{jp} \quad \forall a \in X_{jp};\ \forall b \in Y_{jp}; \quad \forall c \in Z_{jp}\ ..\forall (n-1) \in (N-1)_{jp};\ \forall j \in \mathrm{SN}_p;\ \forall p \in P \tag{17}$$

The flow from a source i in plant p to a 1st level node a that eventually connects to sink j in plant p' $(M^a_{ip,jp'})$ must be equal to the flow associated with the same 1st level node a connecting source i in plant p to any other sink j' in plant p'' $(M^a_{ip,j'p''})$

$$M^a_{ip,jp'} = M^a_{ip,j'p''} \ \forall i \in \mathrm{SU}_p;\ \forall (j,j') \in \mathrm{SN}_p;\ \forall (p,p',p'') \in P;\ \forall a \in X_{ip,jp'} \tag{18}$$

The flow from the freshwater mains to a 1st level node a that eventually connects to sink j in plant p' (F^a_{jp}) must be equal to the flow associated with the same 1st level node a connecting the freshwater mains to any other sink j' in plant p' $(F^a_{j',p'})$

$$F^a_{jp} = F^a_{j',p'} \ \forall (j,j') \in \mathrm{SN}_p;\ \forall (p,p') \in P;\ \forall a \in X_{jp} \tag{19}$$

The flow from a source i in plant p to a 2nd level node b through a 1st level node a that eventually connects to sink j

in plant p' ($M^{a,b}_{ip,jp'}$) must be equal to the flow associated with that 2nd level node b through the same 1st level node a connecting source i in plant p to any other sink j' in plant p'' ($M^{a,b}_{ip,j'p''}$)

$$M^{a,b}_{ip,jp'}=M^{a,b}_{ip,j'p''}\ \ \forall i\in \mathrm{SU}_p;\ \ \forall(j,j')\in \mathrm{SN}_p;\ \ \forall(p,p',p'')\in P;\forall a\in X_{ip,jp'};\ \ \forall b\in Y_{ip,jp'} \tag{20}$$

The flow from the freshwater mains to a 2nd level node b through a 1st level node a that eventually connects to sink j in plant p' ($F^{a,b}_{jp}$) must be equal to the flow associated with that 2nd level node b through the same 1st level node a connecting the freshwater mains to any other sink j' in plant p' ($F^{a,b}_{j',p'}$)

$$F^{a,b}_{jp}=F^{a,b}_{j',p'}\ \ \forall(j,j')\in \mathrm{SN}_p;\ \ \forall(p,p')\in P;\forall a\in X_{jp};\forall b\in Y_{jp} \tag{21}$$

The flow from a source i in plant p to a 3rd level node c through a 2nd level node b and a 1st level node a that eventually connects to sink j in plant p' ($M^{a,b,c}_{ip,jp'}$) must be equal to the flow associated with that 3rd level node c through the same 2nd level node b and 1st level node a connecting source i in plant p to any other sink j' in plant p'' ($M^{a,b,c}_{ip,j'p''}$)

$$M^{a,b,c}_{ip,jp'}=M^{a,b,c}_{ip,j'p''}\ \ \forall i\in SU_p;\ \ \forall(j,j')\in \mathrm{SN}_p;\ \ \forall(p,p',p'')\in P;\forall a\in X_{ip,jp'};\ \ \forall b\in Y_{ip,jp'};\ \ \forall c\in Z_{ip,jp'} \tag{22}$$

The flow from the freshwater mains to a 3rd level node c through a 2nd level node b and a 1st level node a that eventually connects to sink j in plant p' ($F^{a,b,c}_{jp}$) must be equal to the flow associated with that 3rd level node c through the same 2nd level node b and 1st level node a connecting the freshwater mains to any other sink j' in plant p' ($F^{a,b,c}_{j',p'}$)

$$F^{a,b,c}_{jp}=F^{a,b,c}_{j',p'}\ \ \forall(j,j')\in \mathrm{SN}_p;\ \ \forall(p,p')\in P;\forall a\in X_{jp};\ \forall b\in Y_{ip,jp'};\forall c\in Z_{ip,jp'} \tag{23}$$

The flow from a source i in plant p to an nth level node n through an $(n-1)$th level node $(n-1)$ all the way to a 1st level node a that eventually connects to sink j in plant p' ($M^{a,b,c,..,n-1,n}_{ip,jp'}$) must be equal to the flow associated with that nth level node n through the same $(n-1)$th level node $(n-1)$ all the way to the 1st level node a connecting source i in plant p to any other sink j' in plant p'' ($M^{a,b,c,..,n-1,n}_{ip,j'p''}$)

$$M^{a,b,c,..,n-1,n}_{ip,jp'}=M^{a,b,c,..,n-1,n}_{ip,j'p''}\ \ \forall i\in \mathrm{SU}_p;\ \ \forall(j,j')\in \mathrm{SN}_p;\ \ \forall(p,p',p'')\in P;\forall a\in X_{ip,jp'};\ \ \forall b\in Y_{ip,jp'};\ \ \forall c\in Z_{ip,jp'}..\forall(n-1)\in (N-1)_{ip,jp'};\ \ \forall n\in N_{ip,jp'} \tag{24}$$

The flow from the freshwater mains to an nth level node n through an $(n-1)$th level node *(n−1)* all the way to a 1st level node a that eventually connects to sink j in plant p' ($F^{a,b,c,..,n-1,n}_{jp}$) must be equal to the flow associated with that nth level node n through the same $(n-1)$th level node *(n−1)* all the way to the 1st level node a connecting the freshwater mains to any other sink j' in plant p' ($F^{a,b,c,..,n-1,n}_{j',p'}$)

$$F^{a,b,c,..,n-1,n}_{jp}=F^{a,b,c,..,n-1,n}_{j',p'}\ \ \forall(j,j')\in \mathrm{SN}_p;\ \ \forall(p,p')\in P;\ \forall a\in X_{jp};\forall b\in Y_{jp};\forall c\in Z_{jp}\ ..\forall(n-1)\in (N-1)_{jp};\forall n\in N_{jp} \tag{25}$$

The total flows across all branches connecting a source i in plant p to sink j in plant p' must be equal to the individual sum of all flows across each of the branches that establish the connection

$$M^{a}_{ip,jp'}+M^{a,b}_{ip,jp'}+M^{a,b,c}_{ip,jp'}\ +\cdots+\ M^{a,b,c,..,n-1}_{ip,jp'}+M^{a,b,c,..,n}_{ip,jp'}=M_{ip,jp'}\ \ \forall i\in \mathrm{SU}_p;\ \ \forall j\in \mathrm{SN}_p;\ \ \forall p,p'\in P;\forall a\in X_{ip,jp'};\ \forall b\in Y_{ip,jp'};\forall c\in Z_{ip,jp'}..\forall(n-1)\in (N-1)_{ip,jp'};\forall n\in N_{ip,jp'} \tag{26}$$

Similarly, the total flows across all branches connecting the freshwater mains to sink j in plant p' must be equal to the individual sum of all flows across each of the branches that establish the connection

$$F^{a}_{jp}+F^{a,b}_{jp}+M^{a,b,c}_{jp}\ +\cdots+\ F^{a,b,c,..,n-1}_{jp}+F^{a,b,c,..,n}_{jp}=F_{jp}\ \ \forall j\in \mathrm{SN}_p;\ \ \forall p\in P;\forall a\in X_{jp};\forall b\in Y_{jp};\ \forall c\in Z_{jp}..\forall(n-1)\in (N-1)_{jp};\forall n\in N_{jp} \tag{27}$$

Non-negative constraints are required for flows across any branch associated with establishing a connection from source i plant p to sink j plant p'

$$M^{a}_{ip,jp'}\geq 0\ \forall i\in \mathrm{SU}_p;\ \ \forall j\in \mathrm{SN}_p;\ \ \forall p,p'\in P;\forall a\in X_{ip,jp'} \tag{28}$$

$$M^{a,b}_{ip,jp'}\geq 0\ \forall i\in \mathrm{SU}_p;\ \ \forall j\in \mathrm{SN}_p;\ \ \forall p,p'\in P;\forall a\in X_{ip,jp'};\ \forall b\in Y_{ip,jp'} \tag{29}$$

$$M^{a,b,c}_{ip,jp'}\geq 0\ \forall i\in \mathrm{SU}_p;\ \ \forall j\in \mathrm{SN}_p;\ \ \forall p,p'\in P;\forall a\in X_{ip,jp'};\ \forall b\in Y_{ip,jp'};\forall c\in Z_{ip,jp'} \tag{30}$$

$$M^{a,b,c,..,n}_{ip,jp'}\geq 0\ \forall i\in \mathrm{SU}_p;\ \ \forall j\in \mathrm{SN}_p;\ \ \forall p,p'\in P;\forall a\in X_{ip,jp'};\ \forall b\in Y_{ip,jp'};\forall c\in Z_{ip,jp'}..\forall n\in N_{ip,jp'} \tag{31}$$

Similarly, non-negative constraints are required for flows across any branch associated with establishing a connection from the freshwater mains to sink j plant p'

$$F^{a}_{jp}\geq 0\ \forall j\in \mathrm{SN}_p;\ \ \forall p\in P;\ \ \forall a\in X_{jp} \tag{32}$$

$$F^{a,b}_{jp}\geq 0\ \forall j\in \mathrm{SN}_p;\ \ \forall p\in P;\forall a\in X_{jp};\ \ \forall b\in Y_{jp} \tag{33}$$

$$F^{a,b,c}_{jp}\geq 0\ \forall j\in \mathrm{SN}_p;\ \ \forall p\in P;\ \ \forall a\in X_{jp};\forall b\in Y_{jp};\ \ \forall c\in Z_{jp} \tag{34}$$

$$F^{a,b,c,..,n}_{jp}\geq 0\ \forall j\in \mathrm{SN}_p;\ \ \forall p\in P;\forall a\in X_{jp};\ \forall b\in Y_{jp};\ \ \forall c\in Z_{jp}..\forall n\in N_{jp} \tag{35}$$

Backward branching formulation

Equations 36–61 below detail the mathematical formulation associated with a backward branching scheme in a pipeline.

The flow allocated to sink j in plant p' from source i in plant p ($M_{ip,jp'}$) must equal the summation of all flows ($M^{a}_{ip,jp'}$) from the various branches that connects sink j in plant p' to all 1st level nodes a, associated with the stream connection

$$\sum_{a=1}^{X_{ip,jp'}} M^{a}_{ip,jp'} = M_{ip,jp'} \quad \forall i \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p;\ \forall p,p' \in P \quad (36)$$

The flow allocated to wastewater mains from source i in plant p (D_{ip}) must equal the summation of all flows $\left(D^{a}_{ip}\right)$ from the various branches that connect the waste mains to all 1st level nodes a, associated with the stream connection

$$\sum_{a=1}^{X_{ip}} D^{a}_{ip} = D_{ip} \quad \forall i \in \mathrm{SU}_p;\ \forall p \in P \quad (37)$$

The flows allocated from each of the 1st level nodes in the stream that connects sink j in plant p' and source i in plant p $(M^{a}_{ip,jp'})$ must equal the summation of all flows $(M^{a,b}_{ip,jp'})$ from the various branches that connect each 1st level node $a,$ to all 2nd level nodes b associated with the stream connection

$$\sum_{b=1}^{Y_{ip,jp'}} M^{a,b}_{ip,jp'} = M^{a}_{ip,jp'} \quad \forall a \in X_{ip,jp'}; \forall i \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p; \quad \forall p,p' \in P \quad (38)$$

The flows allocated from each of the 1st level nodes in the stream that connects the wastewater mains and source i in plant p (D^{a}_{ip}) must equal the summation of all flows $\left(D^{a,b}_{ip}\right)$ from the various branches that connect each 1st level node $a,$ to all 2nd level nodes b associated with the stream connection

$$\sum_{b=1}^{Y_{ip}} D^{a,b}_{ip} = D^{a}_{ip} \quad \forall a \in X_{ip};\ \forall i \in \mathrm{SU}_p;\ \forall p \in P \quad (39)$$

The flows allocated from each of the 2nd level nodes in the stream that connects sink j in plant p' and source i in plant p $(M^{a,b}_{ip,jp'})$ must equal the summation of all flows $(M^{a,b,c}_{ip,jp'})$ from the various branches that connect each 2nd level node $b,$ to all 3rd level nodes c associated with the stream connection

$$\sum_{c=1}^{Z_{ip,jp'}} M^{a,b,c}_{ip,jp'} = M^{a,b}_{ip,jp'} \quad \forall a \in X_{ip,jp'};\ \forall b \in Y_{ip,jp'};\ \forall i \in \mathrm{SU}_p; \quad \forall j \in \mathrm{SN}_p;\ \forall p,p' \in P \quad (40)$$

The flows allocated from each of the 2nd level nodes in the stream that connects the wastewater mains and source i in plant p $(D^{a,b}_{ip})$ must equal the summation of all flows $\left(D^{a,b,c}_{ip}\right)$ from the various branches that connect each 2nd level node $b,$ to all 3rd level nodes c associated with the stream connection

$$\sum_{c=1}^{Z_{ip}} D^{a,b,c}_{ip} = D^{a,b}_{ip} \quad \forall a \in X_{ip};\ \forall b \in Y_{ip};\ \forall i \in \mathrm{SU}_p;\ \forall p \in P \quad (41)$$

The flows allocated from each of the $(n-1)$th level nodes in the stream that connects sink j in plant p' and source i in plant p $(M^{a,b,c,..,n-1}_{ip,jp'})$ must equal the summation of all flows $(M^{a,b,c,..,n-1,n}_{ip,jp'})$ from the various branches that connect each $(n-1)$th level node, to all nth level nodes associated with the stream connection

$$\sum_{n=1}^{N_{ip,jp'}} M^{a,b,c,..,n-1,n}_{ip,jp'} = M^{a,b,c,..,n-1}_{ip,jp'} \quad \forall a \in X_{ip,jp'}; \forall b \in Y_{ip,jp'}; \quad \forall c \in Z_{ip,jp'} \ ..\forall (n-1) \in (N-1)_{ip,jp'}; \forall i \in \mathrm{SU}_p; \quad \forall j \in \mathrm{SN}_p;\ \forall p,p' \in P \quad (42)$$

The flows allocated from each of the $(n-1)$th level nodes in the stream that connects the wastewater mains and source i in plant p $(D^{a,b,c,..,n-1}_{ip})$ must equal the summation of all flows $\left(D^{a,b,c,..,n-1,n}_{ip}\right)$ from the various branches that connect each $(n-1)$th level node, to all nth level nodes associated with the stream connection

$$\sum_{n=1}^{N_{ip}} D^{a,b,c,..,n-1,n}_{ip} = D^{a,b,c,..,n-1}_{ip} \quad \forall a \in X_{ip};\ \forall b \in Y_{ip}; \quad \forall c \in Z_{ip} \ ..\forall (n-1) \in (N-1)_{ip};\ \forall i \in \mathrm{SU}_p;\ \forall p \in P \quad (43)$$

The flow to sink j in plant p' from a 1st level node a that receives flow from source i in plant p $(M^{a}_{ip,jp'})$ must be equal to the flow associated with the same 1st level node a connecting any other source i' in plant p'' to the same sink j in plant p' $(M^{a}_{i'p'',j'p'})$

$$M^{a}_{ip,jp'} = M^{a}_{i'p'',jp'} \ \forall (i,i') \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p;\ \forall (p,p',p'') \in P; \quad \forall a \in X_{ip,jp'} \quad (44)$$

The flow to the wastewater mains from a 1st level node a that receives flow from source i in plant p (D^{a}_{ip}) must be equal to the flow associated with the same 1st level node connecting any other source i' in plant p' to the waste mains $(D^{a}_{i',p'})$

$$D^{a}_{ip} = D^{a}_{i',p'} \ \forall (i,i') \in \mathrm{SU}_p;\ \forall (p,p') \in P;\ \forall a \in X_{ip} \quad (45)$$

The flow to sink j in plant p' from a 2nd level node b that receives flow through a 1st level node a that eventually connects back to source i in plant p $(M^{a,b}_{ip,jp'},)$ must be equal to the flow associated with that 2nd level node b through the same 1st level node a connecting any other source i' in plant p'' to the same sink j in plant p' $(M^{a,b}_{i'p'',j'p'})$

$$M^{a,b}_{ip,jp'} = M^{a,b}_{i'p'',jp'} \ \forall \left(i,i'\right) \in \mathrm{SU}_p;\ \forall j \in \mathrm{SN}_p;\ \forall (p,p',p'') \in P;\ \forall a \in X_{ip,jp'};\ \forall b \in Y_{ip,jp'} \quad (46)$$

The flow to the wastewater mains from a 2nd level node b that receives flow through a 1st level node a that eventually connects back to source i in plant p $(D^{a,b}_{ip})$ must be equal to the flow associated with that 2nd level node b through the same 1st level node a connecting any other source i' in plant p' to the waste mains $(D^{a,b}_{i',p'})$

$$D^{a,b}_{ip} = D^{a,b}_{i',p'} \ \forall (i,i') \in \mathrm{SU}_p;\ \forall (p,p') \in P;\ \forall a \in X_{ip};\ \forall b \in Y_{ip} \quad (47)$$

The flow to sink j in plant p' from a 3rd level node c that receives flow through a 2nd level node b and a 1st level node a that eventually connects back to source i in plant p $(M^{a,b,c}_{ip,jp'},)$ must be equal to the flow associated with that 3rd level node c through the same 2nd level node b and 1st level node a connecting any other source i' in plant p'' to the same sink j in plant p' $(M^{a,b,c}_{i'p'',j'p'})$

$$M^{a,b,c}_{ip,jp'} = M^{a,b,c}_{i'p'',jp'} \;\; \forall\left(i,i'\right) \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall(p,p',p'') \in P;\;\; \forall a \in X_{ip,jp'};\;\; \forall b \in Y_{ip,jp'};\;\; \forall c \in Z_{ip,jp'} \tag{48}$$

The flow to the wastewater mains from a 3rd level node c that receives flow through a 2nd level node b and a 1st level node a that eventually connects back to source i in plant p ($D^{a,b,c}_{ip}$) must be equal to the flow associated with that 3rd level node c through the same 2nd level node b and 1st level node a connecting any other source i' in plant p' to the waste mains $(D^{a,b,c}_{i',p'})$

$$D^{a,b,c}_{ip} = D^{a,b,c}_{i',p'} \;\; \forall(i,i') \in \mathrm{SU}_p;\;\; \forall(p,p') \in P;\;\; \forall a \in X_{ip};\;\; \forall b \in Y_{ip};\;\; \forall c \in Z_{ip} \tag{49}$$

The flow to sink j in plant p' from an nth level node n through an $(n-1)$th level node $(n-1)$ all the way to a 1st level node a that eventually connects back to source i in plant p ($M^{a,b,c,..,n-1,n}_{ip,jp'}$,) must be equal to the flow associated with that nth level node n through the same $(n-1)$th level node $(n-1)$ all the way to the 1st level node a connecting any other source i' in plant p'' to the same sink j in plant p' $(M^{a,b,c,..,n-1,n}_{i'p'',j'p'})$

$$M^{a,b,c,..,n-1,n}_{ip,jp'} = M^{a,b,c,..,n-1,n}_{i'p'',jp'} \;\; \forall\left(i,i'\right) \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall(p,p',p'') \in P; \forall a \in X_{ip,jp'};\;\; \forall b \in Y_{ip,jp'};\;\; \forall c \in Z_{ip,jp'} .. \forall(n-1) \in (N-1)_{ip,jp'};\;\; \forall n \in N_{ip,jp'} \tag{50}$$

The flow to the wastewater mains from an nth level node n through an $(n-1)$th level node $(n-1)$ all the way to a 1st level node a that eventually connects back to source i in plant p $(D^{a,b,c,..,n-1,n}_{ip})$ must be equal to the flow associated with that nth level node n through the same $(n-1)$th level node $(n-1)$ all the way to the 1st level node a connecting any other source i' in plant p' to the waste mains $(D^{a,b,c,..,n-1,n}_{i',p'})$

$$D^{a,b,c,..,n-1,n}_{ip} = D^{a,b,c,..,n-1,n}_{i',p'} \;\; \forall(i,i') \in \mathrm{SU}_p;\;\; \forall(p,p') \in P;\;\; \forall a \in X_{ip}; \forall b \in Y_{ip}; \forall c \in Z_{ip} \;\; .. \forall(n-1) \in (N-1)_{ip}; \forall n \in N_{ip} \tag{51}$$

The total flows across all branches connecting a source i in plant p to sink j in plant p' must be equal to the individual sum of all flows across each of the branches that establish the connection

$$M^{a}_{ip,jp'} + M^{a,b}_{ip,jp'} + M^{a,b,c}_{ip,jp'} \;+\ldots+\;\; M^{a,b,c,..,n}_{ip,jp'} = M_{ip,jp'} \;\; \forall i \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall p,p' \in P; \forall a \in X_{ip,jp'};\;\; \forall b \in Y_{ip,jp'}; \forall c \in Z_{ip,jp'} .. \forall n \in N_{ip,jp'} \tag{52}$$

Moreover, the total flows across all branches connecting a source i in plant p to the wastewater mains must be equal to the individual sum of all flows across each of the branches that establish the connection

$$D^{a}_{ip} + D^{a,b}_{ip} + D^{a,b,c}_{ip} \;+\ldots+\;\; D^{a,b,c,..,n}_{ip} = D_{ip} \forall i \in SU_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip};\;\; \forall b \in Y_{ip};\;\; \forall c \in Z_{ip} .. \forall n \in N_{ip} \tag{53}$$

Moreover, non-negative constraints are required for flows across any branch associated with establishing a connection from source i plant p to sink j plant p'

$$M^{a}_{ip,jp'} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip,jp'} \tag{54}$$

$$M^{a,b}_{ip,jp'} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip,jp'};\;\; \forall b \in Y_{ip,jp'} \tag{55}$$

$$M^{a,b,c}_{ip,jp'} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip,jp'};\;\; \forall b \in Y_{ip,jp'};\;\; \forall c \in Z_{ip,jp'} \tag{56}$$

$$M^{a,b,c,..,n}_{ip,jp'} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall j \in \mathrm{SN}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip,jp'};\;\; \forall b \in Y_{ip,jp'};\;\; \forall c \in Z_{ip,jp'} .. \forall n \in N_{ip,jp'} \tag{57}$$

Similarly, non-negative constraints are required for flows across any branch associated with establishing a connection from source i plant p to the wastewater mains

$$D^{a}_{ip} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall i \in \mathrm{SU}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip} \tag{58}$$

$$D^{a,b}_{ip} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall i \in \mathrm{SU}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip};\;\; \forall b \in Y_{ip} \tag{59}$$

$$D^{a,b,c}_{ip} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall i \in \mathrm{SU}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip};\;\; \forall b \in Y_{ip};\;\; \forall c \in Z_{ip} \tag{60}$$

$$D^{a,b,c,..,n}_{ip} \geq 0 \;\; \forall i \in \mathrm{SU}_p;\;\; \forall i \in \mathrm{SU}_p;\;\; \forall p \in P;\;\; \forall a \in X_{ip};\;\; \forall b \in Y_{ip};\;\; \forall c \in Z_{ip} .. \forall n \in N_{ip} \tag{61}$$

Problem implementation

Since all source-to-sink connectivity options can take on both forms of branching, two different NLP optimization problems were solved in this work: (a) applying the forward merging formulation for source-to-sink and fresh-to-source connectivity (Eqs. 10–35) and (b) applying the backward merging formulation for the source-to-sink and sink-to-waste connectivity (Eqs. 36–61). Both problems were implemented using "what'sBest9.0.5.0" LINDO Global Solver for Microsoft Excel 2010 on a desktop PC (Intel® Core™ i7–2620M, 2.7 GHz, 8.00 GB RAM, 64-bit Operating System).[34]

Case Study

To demonstrate the pipeline merging aspects that have been accounted for in interplant water network synthesis problems, an illustrative case study example has been carried out as an illustration. The case study is adopted from Alnouri et al.,[22] which considers each source-sink connection to be a separate pipeline. We have solved the two different problem formulations separately so as to compare the differences between applying forward and backward branching for the source-to-sink connectivity. The aim of this case study is to illustrate that merged networks can outperform segregated networks and are therefore important to consider in optimal interplant water integration. It was observed that merged pipelines offer more attractive solutions in terms of overall network cost-efficiency when compared to solutions attained when utilizing a single pipeline for each allocation involved within the network.

Figure 7 shows the industrial city layout that has been considered, which consists of an arrangement of six different industrial facility entities, six water sources, and six water sinks distributed across the cluster of plants. The plot was assumed to have a total area of 64 km^2. A case study that involves the same arrangement of plants has been previously

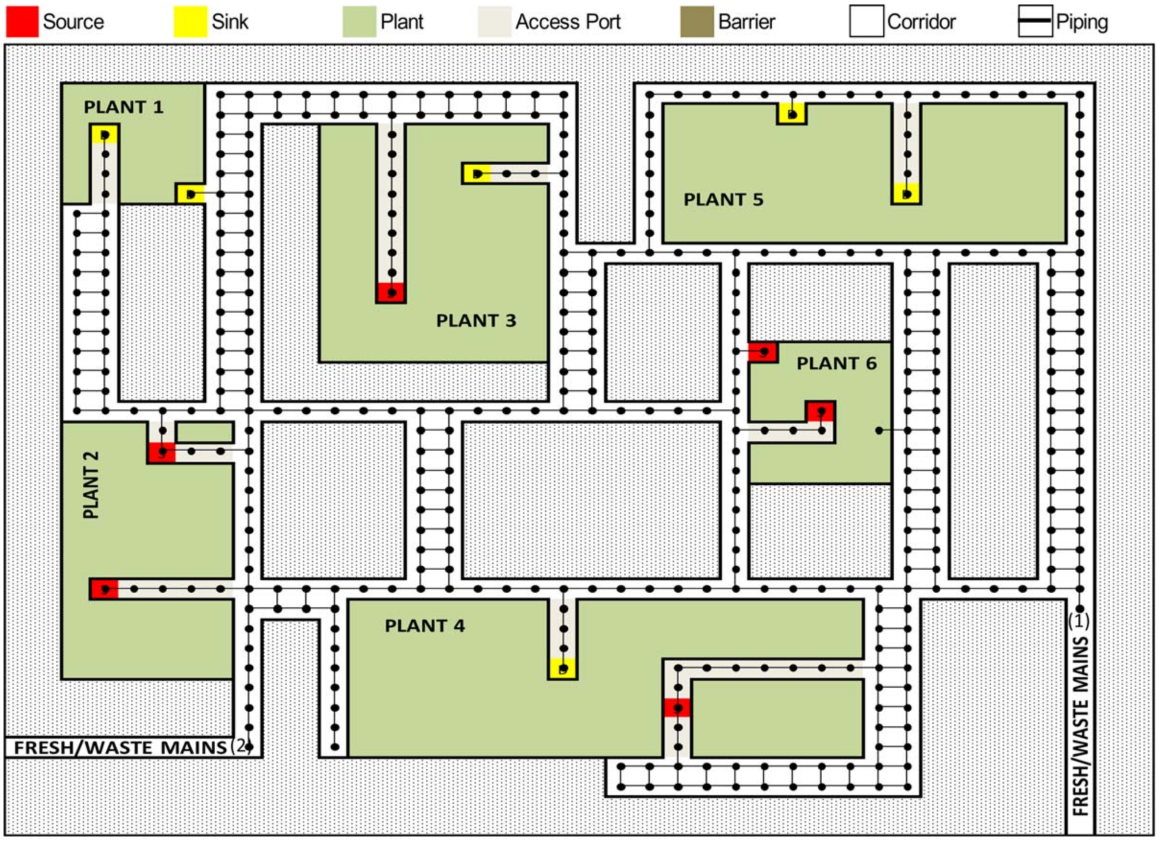

Figure 7. Industrial Zone arrangement for Case Study.[22]

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

implemented, using a separate pipeline for every water allocation achieved.[22] In this work, results from both the previous and current implementation will be compared, so as to identify the best performing scenarios in terms of pipeline assembling options. Two interchanging locations have been assumed for the freshwater supply and the wastewater discharge mains, as illustrated in Figure 7. This helps in examining the influence of altering their respective positions on the piping arrangements attained, as well as the overall networks costs achieved. For each of these two cases, both forward and backward branching scenarios are applied on all source-to-sink connectivity within the network. Two different scenarios of merged pipeline instances, for source-to-sink interplant water transmission were studied. Thus, a total of four different options have been considered for the case study: (a) Case 1-forward branching on all source-to-sink connectivity, with Position 1 for the fresh mains and Position 2 for the waste mains; (b) Case 2-forward branching on all source-to-sink connectivity, with Position 2 for the fresh mains and Position 1 for the waste mains; (c) Case 3-backward branching on all source-to-sink connectivity, with Position 1 for the fresh mains and Position 2 for the waste mains; and (d) Case 4-backward branching on all source-to-sink connectivity, with Position 2 for the fresh mains and Position 1 for the waste mains. Based on the explanation provided in the methodology section of this article, it should be noted that only forward branching was implemented on the freshwater mains, and only backward branching was implemented for the wastewater mains in the various cases described earlier, even though both types of branching arrangements were investigated for source-to-sink connectivity involved.

Extracting the various optimum routing options, as well as the shortest path lengths associated with the respective pipeline branches was carried out using an analogous approach to the methodology that has been introduced in earlier work.[22] In this work, only Type 1 connectivity was used for illustration purposes. Hence, a single connectivity mesh was developed for extracting optimum routing in right-angled

Table 1. Multiple Contaminant Source Data

Water Sources	Flow (kg/h)	Conc. X1 (ppm)	Conc. X2 (ppm)	Conc. X3 (ppm)
P2S2	120,000	100	50	30
P2S1	80,000	140	100	60
P3S1	140,000	180	150	130
P6S2	80,000	230	180	180
P6S1	195,000	250	190	200
P4S1	100,000	100	190	210

Table 2. Multiple Contaminant Sink Data

Water Sinks	Flow (kg/h)	Max. Inlet Conc. X1 (ppm)	Max. Inlet Conc. X2 (ppm)	Max. Inlet Conc. X3 (ppm)
P1D1	120,000	0	0	30
P1D2	80,000	50	50	80
P3D1	80,000	50	70	100
P5D1	140,000	140	100	100
P5D2	80,000	170	120	130
P4D1	195,000	240	130	150

Table 3. Optimized Distances (km) and Flows (t/h) Associated with Forward Branching, Using Multiple Contaminant Information (Case 1)

P2S1	0.4	0	N	3.2	0	P1D1												
			N	2.8	0	P1D2												
	0.6	80	N	1.4	0	N	1.6	0	WASTE									
			N			N	3	0	P4D1									
			N	2.6	80	N	3	0	P3D2									
			N			N	4.8	80	N	0.2	32	P5D1						
			N			N			N	1.8	48	P5D2						
P2S2	1	120	N	1.6	0	WASTE												
			N	2.6	25.7	N	3	0	P1D1									
			N			N	1.8	25.71	P1D2									
			N	1.4	94.2	N	1.6	66.26	P4D1									
			N			N	2.6	28.02	N	3	14.28	P3D2						
			N			N			N	4.8	13.73	N	0.2	13.7	P5D1			
			N			N			N			N	1.8	0	P5D2			
P3S1	1.8	140	N	1.8	0	N	0.4	0	P1D2									
			N			N	2.2	0	N	3.8	0	P1D1						
			N			N			N	3.4	0	WASTE						
			N	1.8	140	N	0.6	0	P3D2									
			N			N	0.8	140	N	5.8	40.57	P4D1						
			N			N			N	3.2	99.42	N	0.2	67.42	P5D1			
			N			N			N			N	1.8	32	P5D2			
P6S1	0.2	195	N	0.6	195	N	3.6	0	N	3.6	0	P1D1						
			N			N			N	2.4	0	P1D2						
			N			N	3	195	N	3.8	186.8	WASTE						
			N			N			N	0.8	8.15	P4D1						
			N	1.6	0	N		0	P3D2									
			N			N	2.6	0	N	0.2	0	P5D1						
			N			N			N	1.8	0	P5D2						
P6S2	0.8	80	N	0.2	0	N	1.2	0	N	3	0	P3D2						
			N			N			N	2.4	0	N	2.4	0	P1D2			
			N			N			N			N	3.6	0	P1D1			
			N			N	4.8	0	N	0.2	0	P5D1						
			N			N			N	1.8	0	P5D2						
			N	2.8	80	N	0.8	80	P4D1									
			N			N	3.8	0	WASTE									
P4S1	1.8	100	N	9.4	0	P5D2												
			N	1.8	100	N	1.2	74.2	N	0.8	0	P4D1						
			N			N			N	1.6	74.28	N	2.2	60	WASTE			
			N			N			N			N	2	14.28	N	4.2	0	P1D1
			N			N			N			N			N	4.2	14.28	P1D2
			N			N	1.8	25.71	N	4.2	25.71	P3D2						
			N			N			N	5	0	P5D1						
FRESH	0.2	226.8421	N	2.4	200	N	6	40	P3D2									
			N			N	1.2	160	N	0.8	0	P4D1						
			N			N			N	1.6	160	N	2.2	0	WASTE			
			N			N			N			N	2	160	N	4.2	120	P1D1
			N			N			N			N			N	3	40	P1D2
			N	6.2	26.84	N	1	26.84	P5D1									
			N			N	1	0	P5D2									

Table 4. Diameters (m) Associated with Forward Branching, Using Multiple Contaminant Information (Case 1)

P2S1	0	0	N	0	0	P1D1												
			N	0	0	P1D2												
	0.160	0.2	N	0	0	N	0	0	WASTE									
			N			N	0	0	P4D1									
			N	0.160	0.2	N	0	0	P3D1									
			N			N	0.160	0.2	N	0.106	0.2	P5D1						
			N			N			N	0.127	0.2	P5D2						
P2S2	0.192	0.2	N	0	0	WASTE												
			N	0.096	0.1	N	0	0	P1D1									
			N			N	0.096	0.1	P1D2									
			N	0.173	0.2	N	0.147	0.2	P4D1									
			N			N	0.101	0.2	N	0.074	0.1	P3D1						
			N			N			N	0.072	0.1	N	0.072	0.1	P5D1			
			N			N			N			N	0	0	P5D2			
P3S1	0.206	0.3	N	0	0	N	0	0	P1D2									
			N			N	0	0	N	0	0	P1D1						
			N			N			N	0	0	WASTE						
			N	0.206	0.3	N	0	0	P3D1									
			N			N	0.206	0.3	N	0.118	0.2	P4D1						
			N			N			N	0.177	0.2	N	0.148	0.2	P5D1			
			N			N			N			N	0.106	0.2	P5D2			
P6S1	0.239	0.3	N	0.239	0.3	N	0	0	N	0	0	P1D1						
			N			N			N	0	0	P1D2						
			N			N	0.231	0.3	N	0.235	0.3	WASTE						
			N			N			N	0.057	0.1	P4D1						
			N	0	0	N	0	0	P3D1									
			N			N	0	0	N	0	0	P5D1						
			N			N			N	0	0	P5D2						
P6S2	0.160	0.2	N	0	0	N	0	0	N	0	0	P3D1						
			N			N			N	0	0	N	0	0	P1D2			
			N			N			N			N	0	0	P1D1			
			N			N	0	0	N	0	0	P5D1						
			N			N			N	0	0	P5D2						
			N	0.160	0.2	N	0.160	0.2	P4D1									
			N			N	0	0	WASTE									
P4S1	0.177	0.2	N	0	0	P5D2												
			N	0.177	0.2	N	0.155	0.2	N	0	0	P4D1						
			N			N			N	0.155	0.2	N	0.141	0.2	WASTE			
			N			N			N			N	0.074	0.1	N	0	0	P1D1
			N			N			N			N			N	0.074	0.1	P1D2
			N			N	0.0964	0.1	N	0.096	0.1	P3D1						
			N			N			N	0	0	P5D1						
FRESH	0.256	0.3	N	0.242	0.3	N	0.117	0.2	P3D1									
			N			N	0.219	0.3	N	0	0	P4D1						
			N			N			N	0.219	0.3	N	0	0	WASTE			
			N			N			N			N	0.219	0.3	N	0.192	0.2	P1D1
			N			N			N			N			N	0.117	0.2	P1D2
			N	0.098	0.1	N	0.098	0.1	P5D1									
			N			N	0	0	P5D2									

Table 5. Optimized Distances (km) and Flows (t/h) Associated with Backward Branching, Using Multiple Contaminant Information (Case 3)

P1D1	3.2	120	N	0.4	0	P2S1															
			N	0.6	120	N	2.8	0	P2S2												
			N			N	5.8	0	P3S1												
			N			N	3.4	120	N	0.8	0	P6S1									
			N			N			N	0.2	120	N	0.8	0	P6S2						
			N			N			N			N	2.6	120	N	2.6	0	P4S1			
			N			N			N			N			N	1.6	120	FRESH			
P1D2	0.2	80	N	0.6	0	P2S1															
			N	0.2	80	N	3.6	0	P3S1												
			N			N	2.2	80	N	2.8	25.71	P2S2									
			N			N			N	3.4	54.28	N	0.8	0	P6S1						
			N			N			N			N	0.2	54.28	N	0.8	0	P6S2			
			N			N			N			N			N	2.6	54.28	N	1.6	40	FRESH
			N			N			N			N			N			N	1.6	14.28	P4S1
P3D1	0.6	80	N	3.6	0	P3S1															
			N	0.8	80	N	2.4	14.28	N	2.4	0	P2S1									
			N			N			N	4.2	14.28	P2S2									
			N			N	1.2	65.71	N	6	40	FRESH									
			N			N			N	1	25.71	N	0.2	0	P6S1						
			N			N			N			N	0.8	25.71	N	0.8	0	P6S2			
			N			N			N			N			N	5.2	25.71	P4S1			
P4D1	0.8	195	N	9.4	40.57	P3S1															
			N	1.2	88.15	N	1.6	88.15	N	1	8.157	P6S1									
			N			N			N	0.8	80	P6S2									
			N			N	1	0	N	2.6	0	P4S1									
			N			N			N	1.6	0	FRESH									
			N	0.8	66.26	N	1.4	66.26	N	2	0	P2S1									
			N			N			N	1	66.26	P2S2									
P5D1	0.2	140	N	7.2	26.84	FRESH															
			N	2.6	113.1	N	0.4	113.1	N	4.6	67.42	P3S1									
			N			N			N	2.6	45.73	N	2.4	32	P2S1						
			N			N			N			N	4.2	13.73	P2S2						
			N			N	1.6	0	N	0.2	0	P6S1									
			N			N			N	0.8	0	N	0.8	0	P6S2						
			N			N			N			N	5.2	0	P4S1						
P5D2	1	80	N	3.4	80	N	0.4	80	N	4.6	32	P3S1									
			N			N			N	2.6	48	N	2.4	48	P2S1						
			N			N			N			N	4.2	0	P2S2						
			N			N	1.6	0	N	0.2	0	P6S1									
			N			N			N	1.6	0	P6S2									
			N	6.2	0	N	0.2	0	FRESH												
			N			N	4	0	P4S1												
WASTE	1.4	246.84	N	0.2	0	N	1	0	P2S2												
			N			N	1.4	0	N	0.6	0	P2S1									
			N			N			N	6.2	0	P3S1									
			N	3.6	246.84	N	1.6	186.84	N	1	186.8	P6S1									
			N			N			N	0.8	0	P6S2									
			N			N	1	60	N	2.6	60	P4S1									
			N			N			N	1.6	0	FRESH									

Table 6. Diameters (m) Associated With Backward Branching, Using Multiple Contaminant information (Case 3)

P1D1	0.192	0.2	N	0	0	P2S1															
			N	0.192	0.2	N	0	0	P2S2												
			N			N	0	0	P3S1												
			N			N	0.192	0.2	N	0	0	P6S1									
			N			N			N	0.192	0.2	N	0	0	P6S2						
			N			N			N			N	0.192	0.2	N	0	0	P4S1			
			N			N			N			N			N	0.192	0.2	FRESH			
P1D2	0.160	0.2	N	0	0	P2S1															
			N	0.160	0.2	N	0	0	P3S1												
			N			N	0.160	0.2	N	0.096	0.1	P2S2									
			N			N			N	0.134	0.2	N	0	0	P6S1						
			N			N			N			N	0.134	0.2	N	0	0	P6S2			
			N			N			N			N			N	0.134	0.2	N	0.117	0.2	FRESH
			N			N			N			N			N			N	0.074	0.1	P4S1
P3D1	0.160	0.2	N	0	0	P3S1															
			N	0.160	0.2	N	0.074	0.1	N	0	0	P2S1									
			N			N			N	0.074	0.1	P2S2									
			N			N	0.147	0.2	N	0.117	0.2	FRESH									
			N			N			N	0.096	0.1	N	0	0	P6S1						
			N			N			N			N	0.096	0.1	N	0	0	P6S2			
			N			N			N			N			N	0.096	0.1	P4S1			
P4D1	0.239	0.3	N	0.118	0.2	P3S1															
			N	0.167	0.2	N	0.167	0.2	N	0.057	0.1	P6S1									
			N			N			N	0.160	0.2	P6S2									
			N			N	0	0	N	0	0	P4S1									
			N			N			N	0	0	FRESH									
			N	0.147	0.2	N	0.147	0.2	N	0	0	P2S1									
			N			N			N	0.147	0.2	P2S2									
P5D1	0.206	0.3	N	0.098	0.1	FRESH															
			N	0.187	0.2	N	0.187	0.2	N	0.148	0.2	P3S1									
			N			N			N	0.124	0.2	N	0.106	0.2	P2S1						
			N			N			N			N	0.072	0.1	P2S2						
			N			N	0	0	N	0	0	P6S1									
			N			N			N	0	0	N	0	0	P6S2						
			N			N			N			N	0	0	P4S1						
P5D2	0.160	0.2	N	0.16	0.2	N	0.16	0.2	N	0.106	0.2	P3S1									
			N			N			N	0.127	0.2	N	0.127	0.2	P2S1						
			N			N			N			N	0	0	P2S2						
			N			N	0	0	N	0	0	P6S1									
			N			N			N	0	0	P6S2									
			N	0	0	N	0	0	FRESH												
			N			N	0	0	P4S1												
WASTE	0.266	0.3	N	0	0	N	0	0	P2S2												
			N			N	0	0	N	0	0	P2S1									
			N			N			N	0	0	P3S1									
			N	0.266	0.3	N	0.235	0.3	N	0.235	0.3	P6S1									
			N			N			N	0	0	P6S2									
			N			N	0.141	0.2	N	0.141	0.2	P4S1									
			N			N			N	0	0	FRESH									

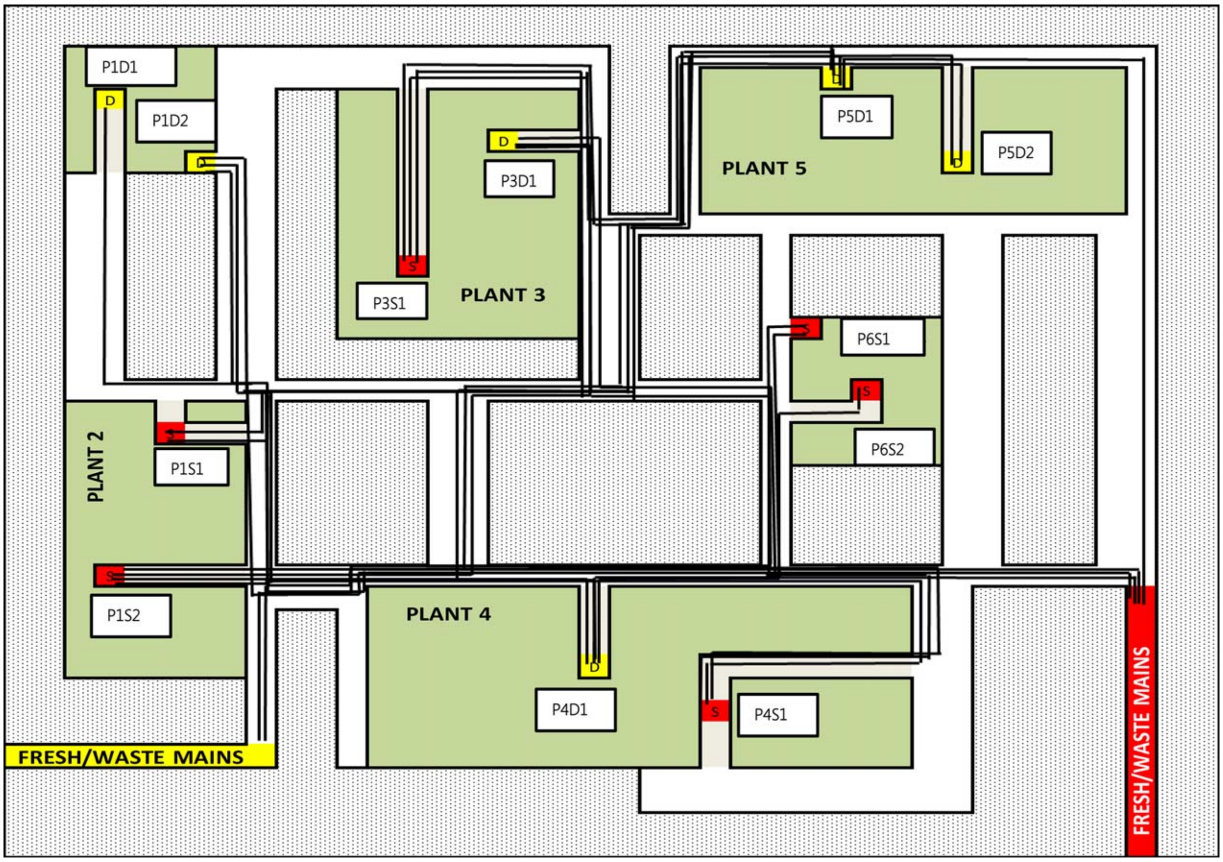

Figure 8. Case 1 unmerged interplant network solution illustrated.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

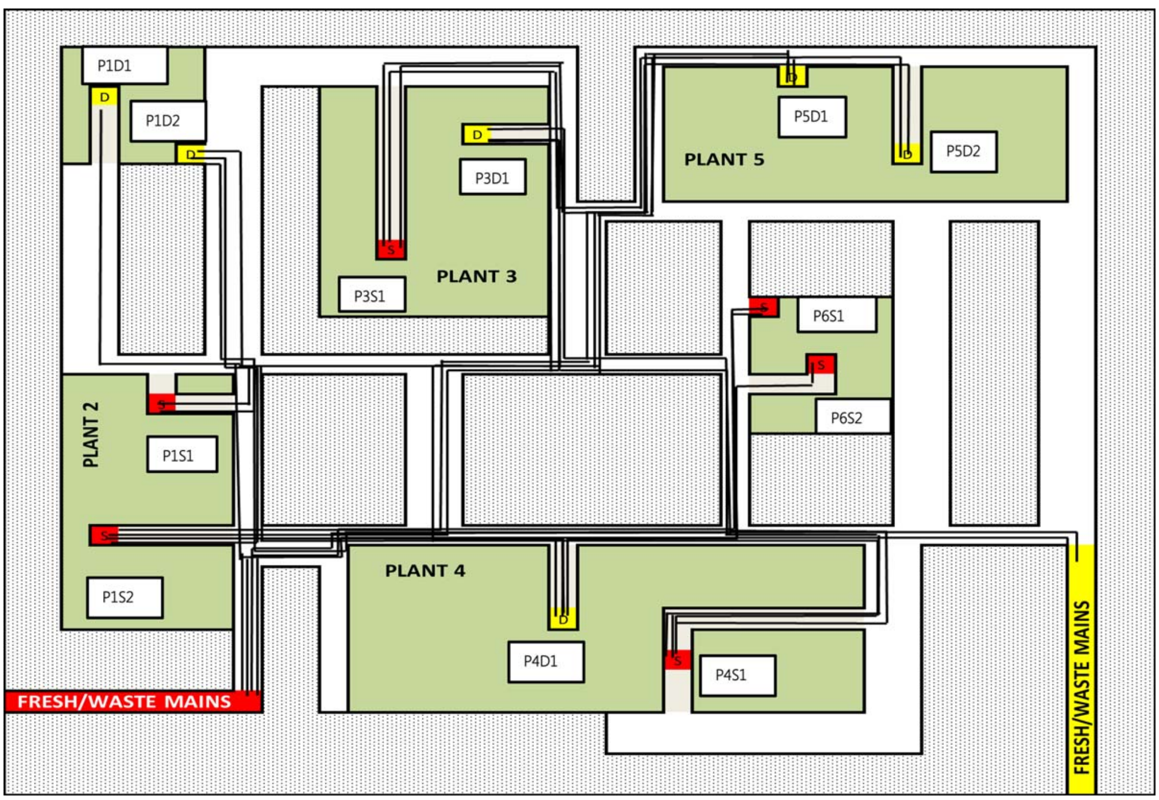

Figure 9. Case 2 unmerged interplant network solution illustrated.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

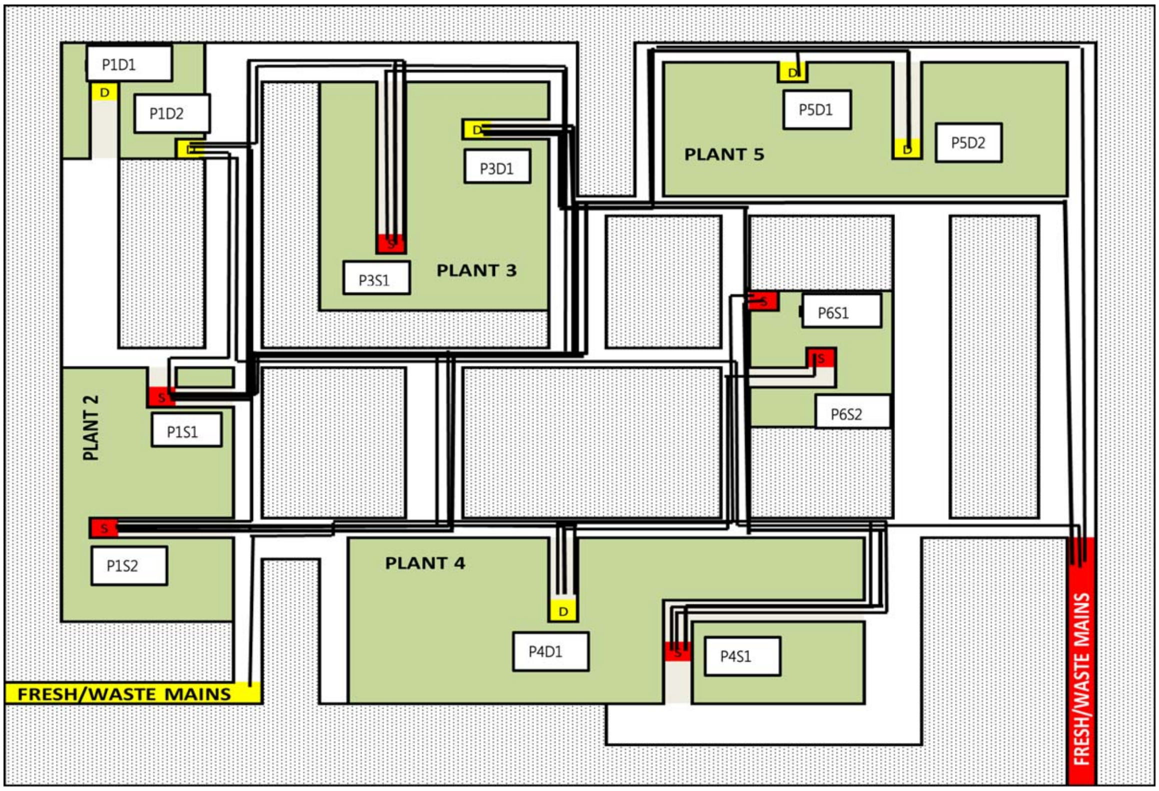

Figure 10. Case 3 unmerged interplant network solution illustrated.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

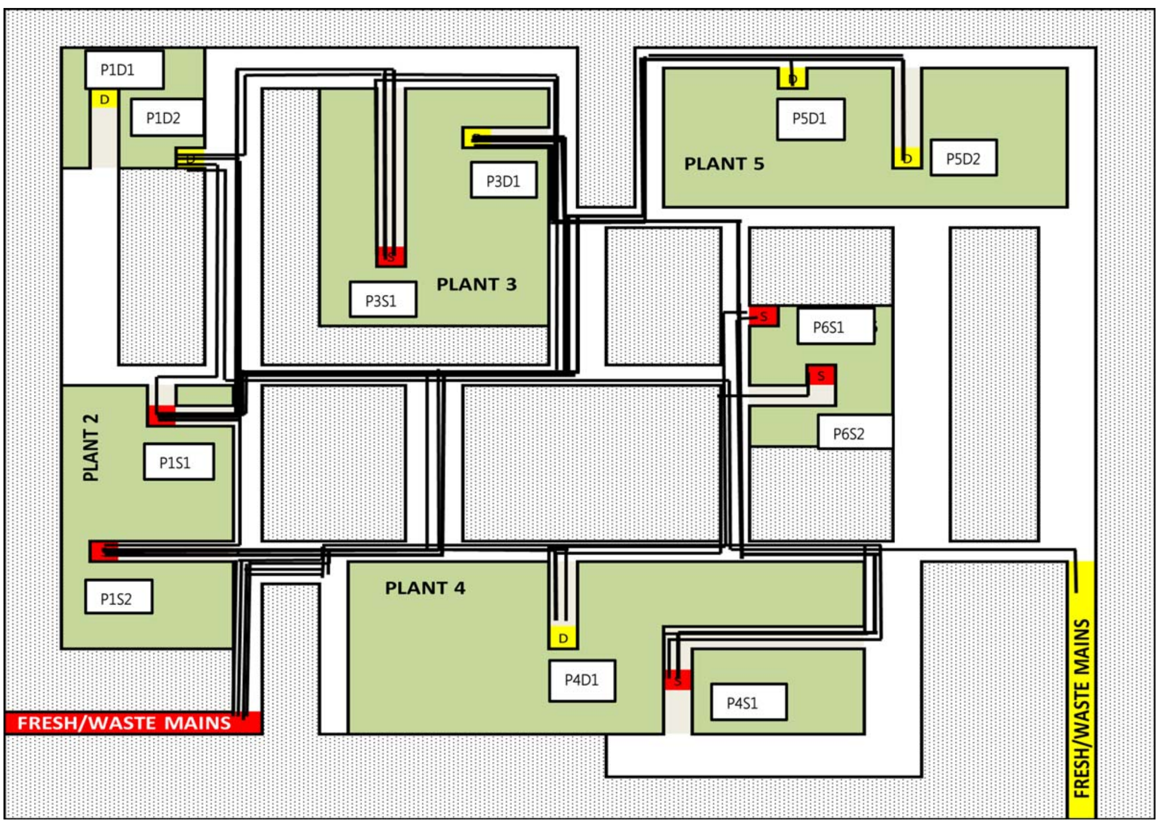

Figure 11. Case 4 unmerged interplant network solution illustrated.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

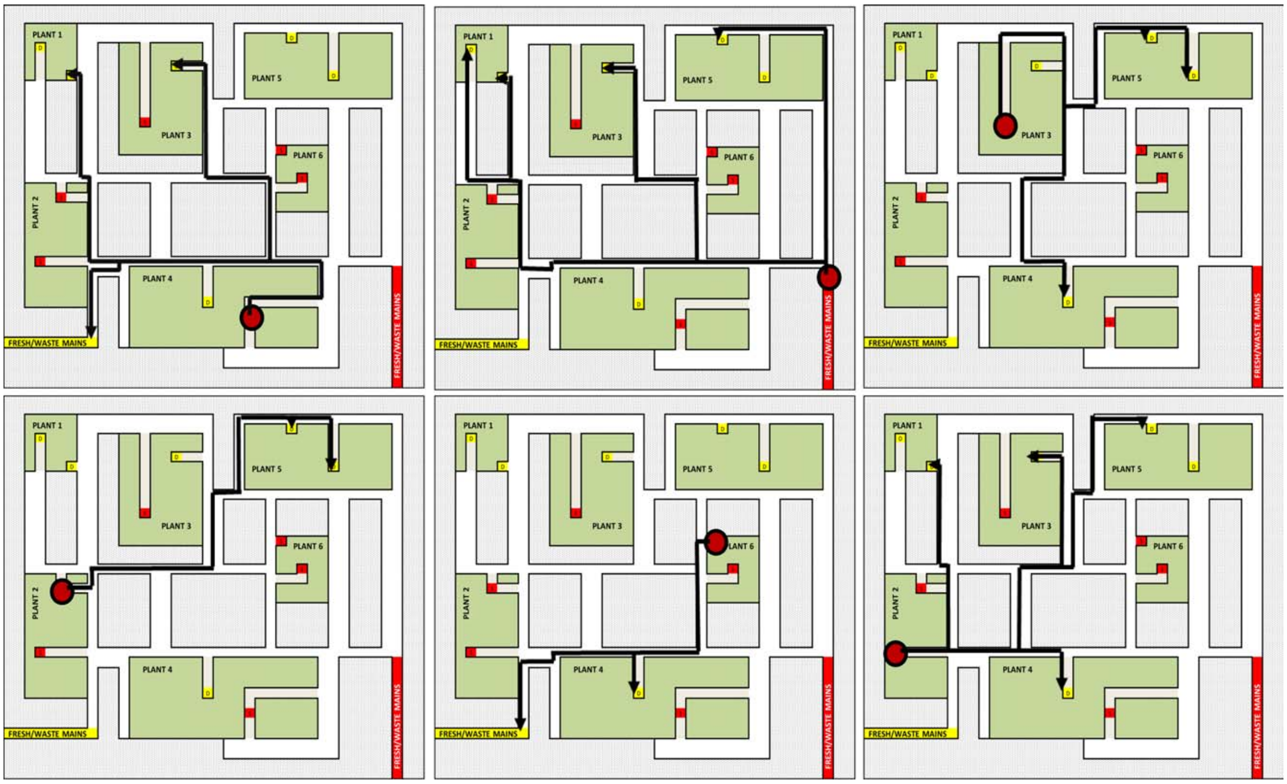

Figure 12. Case 1 interplant piping illustrated after merging, via forward branching.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

pathways.[22] All cases were carried out using multiple contaminant information, while implementing all the four different settings that have been described above. Water source and sink flows, as well as source and sink contaminant information utilized in each of the different cases, are provided in Tables 1 and 2, respectively. Carbon steel Schedule 80 welded pipes, with cost parameters $a = 696.58$ and $b = 1.215$, were used for all cases.[22] Moreover, a freshwater

Figure 13. Case 2 interplant piping illustrated after merging, via forward branching.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

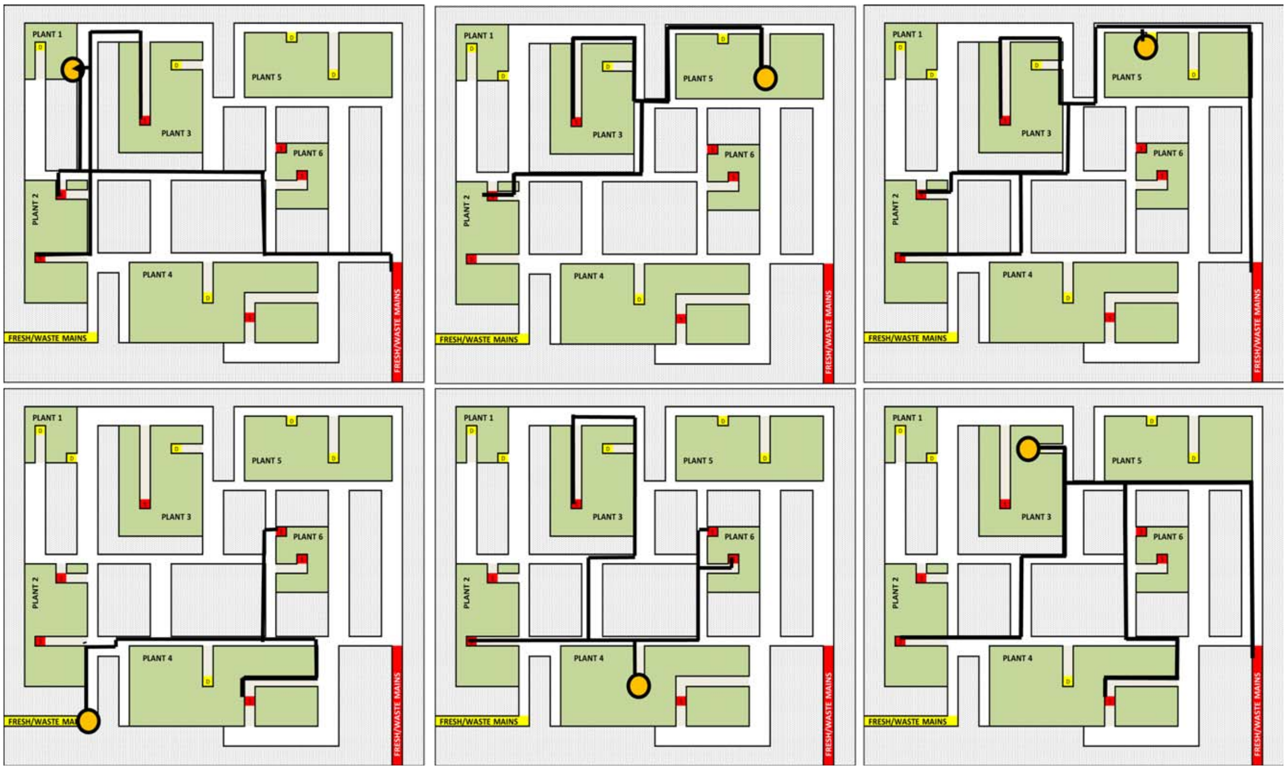

Figure 14. Case 3 interplant piping illustrated after merging, via backward branching.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

cost (C^{FRESH}) of 0.13 \$/ton was utilized, in addition to assuming 8760 h/yr of operating hours (H_y). Additionally, all capital expenses were annualized using a constant factor $(\gamma) = 0.05$.

When minimizing the total network costs for the different cases in terms of merged pipeline expenses as well as freshwater consumption, a total of 226.8 and 246.8 t/h of minimum freshwater use and wastewater discharge were

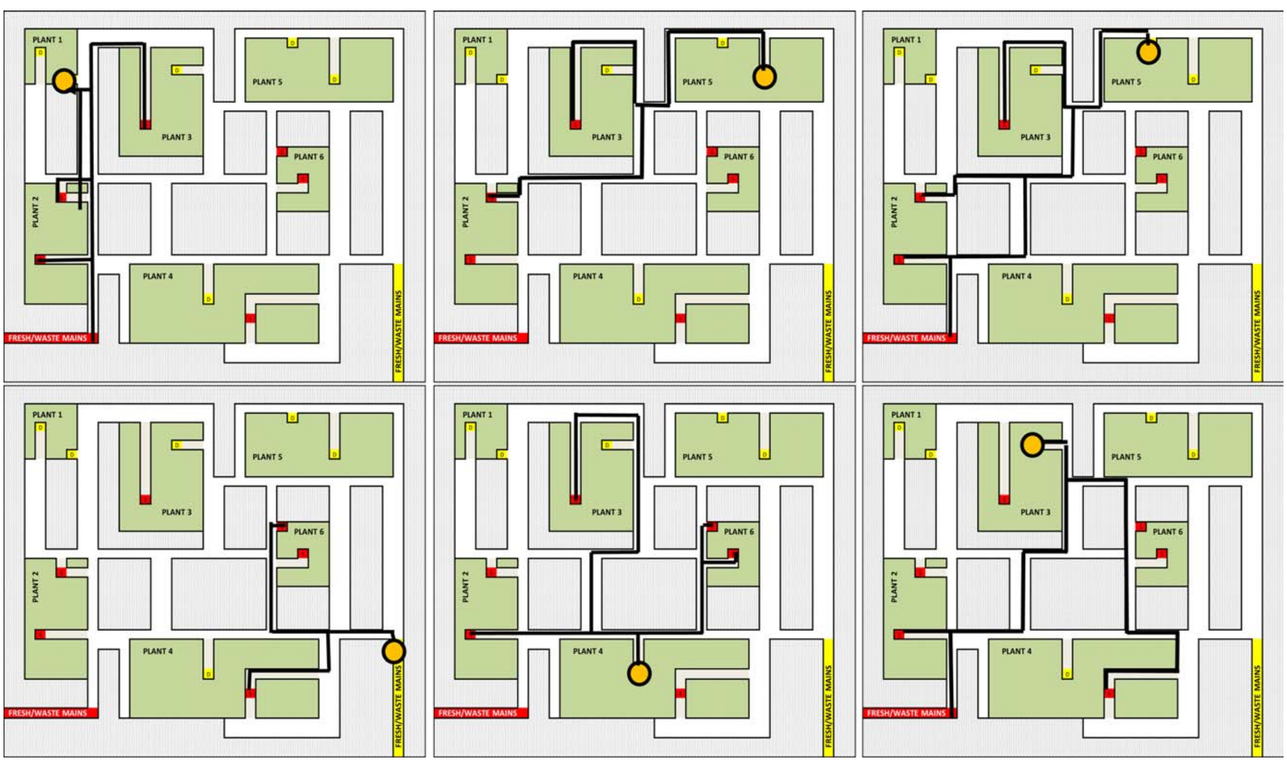

Figure 15. Case 4 interplant piping illustrated after merging, via backward branching.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 7. Cost Summary of All Scenarios Investigated with Pipeline Merging and a Comparison of the Network Cost Obtained Before and After Pipeline Merging

Cost Item	Forward Branching Case 1	Forward Branching Case 2	Backward Branching Case 3	Backward Branching Case 4
Pipeline costs ($)	12,011,167	11,655,738	12,562,751	9,954,339
Total fresh costs ($/yr)	258,328	258,328	258,328	258,328
Annualized piping + fresh costs ($/yr)	858,886	841,115	886,465	756,045
% Savings	−4.193% Savings	+2.713 (No savings)	−1.117% Savings	−7.675 Savings

achieved respectively, for all the different scenarios that have been investigated (Cases 1–4). All source-sink mapping implementations that have been obtained were found to satisfy the same target values of minimum fresh and waste. Table 3 summarizes all optimized pipeline branch lengths using a forward branching scenario, as well as provides the values of the water flow rates associated with each branch, for Case 1. For that same case, Table 4 lists all the pipeline diameters that were obtained for each branch. Table 5 conversely summarizes all optimized pipeline branch lengths using a backward branching scenario (Case 3), as well as provides the values for all water flow rates associated with each branch. Table 6 provides all the pipeline diameters that were obtained for each branch. According to the results obtained, interchanging the freshwater and wastewater mains positions had no effect on the implementation obtained, neither on the diameters of the respective branches within the implementation. The only values changed were the optimized pipeline branch lengths associated with the fresh mains supplying water to the different sinks (i.e., the forward branching—Case 2), as well as the pipe branch lengths associated with waste mains receiving water from the various sources (i.e., the backward branching—Case 4).

Figures 8, 9, 10, and 11 provide illustrations of unmerged interplant network connectivity for Cases 1–4, respectively, utilizing the shortest routing options within the boundaries of the industrial city arrangement that has been provided. In all cases, many pipeline connections were attained in the optimal solution, thus indicating that it would be quite difficult to track and manage all pipeline transmission implementations attained. Figures 12, 13, 14, and 15 illustrate the corresponding merged pipeline solutions attained for the various interplant network designs. Figure 12 provides schematics of each optimal merged pipeline schematics via forward branching, for each given water source, distributing water to all sinks involved, while assuming Position 1 for the fresh mains and Position 2 for the waste mains. Figure 13 illustrates the different pipeline merging schematics via forward branching, when the fresh and waste mains positions are interchanged. It should be noted that the only single unmerged pipeline was associated with water source 2 in plant 6, transmitting water to sink 1 in plant 4, and hence was not shown in Figures 12 and 13. As aforementioned in this section, both forward and backward branching schemes, were investigated. Figure 14 illustrates the different pipeline merging schematics for all connections via backward branching, for each water sink, receiving water from all sources involved, while assuming Position 1 for the fresh mains and Position 2 for the waste mains. Similarly, Figure 15 illustrates the different pipeline merging schematics via backward branching, when the fresh and waste mains positions are interchanged. Similar to the forward branching cases, it should be noted that the only single unmerged pipeline was associated with freshwater being delivered to water source 1 in plant 1, and hence was not shown in Figures 14 and 15. Based on the solutions attained, it was evident that both forward and backward branching scenarios, the pipeline schematics do change according to the two different locations for the mains that have been assumed, as well as according to the branching scheme involved.

The respective network costs attained for the different scenarios that have been investigated are summarized in Table 7. The results indicate that forward branching was found to be more economical than backward branching in some cases, and vice versa, depending on the fresh and waste positions that have been assumed on the plot. For instance, forward branching was found to yield more cost effective solutions when compared to backward branching, assuming Position 1 for the fresh mains and Position 2 for the waste mains. Conversely, when Position 2 was assumed for the fresh mains and Position 1 was assumed for the waste mains, backward pipeline branching gave more attractive solutions.

The annualized piping costs that were obtained when no merging in between pipelines was implemented were all taken from previous work,[22] are had the following values: $896,478/yr for the case assuming Position 1 for the fresh mains and Position 2 for the waste mains, that was compared with Cases 1 and 3 of this article, and $818,898/yr assuming Position 2 for the fresh mains and Position 4 for the waste mains, that was compared with Cases 2 and 4 of this article. When assessed against the current results, after implementing the various pipeline merging scenarios that have been discussed, it was found that some of the merged cases do yield savings in terms of the piping costs obtained for the network. All savings were calculated accordingly, and provided in Table 7. It was observed that backward branching allowed for more savings in terms of network costs, compared to forward branching, with Case 4 being the highest in overall savings. Moreover, the results show that Case 2 incurs slight additional expenses after implementing pipeline merging schemes. This case resulted in no savings achieved, which was attributed to the fact that no extra flow was added to already existing pipeline diameters. The corresponding pipeline diameters utilized after merging had to be substantially increased, so as to accommodate for the combined water flow rate values to be transmitted and distributed within the network.

Conclusions

Interplant water integration often entails the use of methodologies that could provide insight into how much freshwater consumption and wastewater discharge can be minimized to reach their respective targets, so as to allow for maximized water reuse amongst the various processing industries. This work investigates opportunities for carrying out interplant water network synthesis, while implementing pipeline merging arrangements within the designs, for water allocation, transmission, and distribution amongst a given arrangement of plants within an industrial zone. For the purpose of attaining merged pipeline implementations, two different pipeline branching schemes were carried out in this work, forward branching, and backward branching. An illustrative case study has been carried out to demonstrate the proposed methodology, in which both different branching scenarios were investigated, using multiple contaminant information.

We have presented the first approach to address pipeline merging to water network synthesis. The main motivation has been to highlight that merged pipeline options can offer cost as well as complexity advantages over the standard assumption of segregated pipe connections between sources and sinks. The proposed scheme of pipe merging is not exhaustive and other merged pipeline options may exist that offer benefits. Future work will further develop the representation toward the inclusion of larger numbers of option.

For the two different formulations were adopted for the branching schemes, the type of branching utilized for all connections associated with each of the connectivity categories, that is, (1) source-to-sink, (2) fresh-to-source, and (3) sink-to-waste, has been assumed to be the same in each case. As mentioned in the methodology discussion, connectivity categories (2) and (3) can only involve one of the branching types. However, source-to-sink connectivity has been allowed to incorporate a mix of both options. The case study illustrates the application of each branching scheme separately and does not combine more than one merging choice for source-to-sink connectivity. However, there could be options in which a certain degree of mixing between forward and backward branching within the same connectivity category, that can outperform a single branching scheme solution. As a potential extension to this work, this aspect could be further investigated. Additionally, other merging options can be further investigated in terms of incorporating water quality specifications for interplant water transfer, which may be less efficient in terms of water use due to stream mixing, but could possibly lead to more efficient designs in terms of infrastructure cost. These aspects will be addressed in subsequent publications.

Acknowledgments

This publication was made possible by NPRP grant no. 4-1191-2-468 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

Notation

Indices

p = plant/process
i = water source
j = water sink
c = contaminant
a = first level node associated with pipeline branching
b = second level node associated with pipeline branching
c = third level node associated with pipeline branching
n = Nth level node associated with pipeline branching

Sets

P = set of plants/processes in industrial city
SU_p = set of water sources in plant p
SN_p = set of water sinks in plant p
C = set of contaminants/pollutants
$X_{ip,jp'}$ = set of 1st level nodes associated with stream connecting source i plant p to sink j plant p' using either a forward or backward branching scenario
$Y_{ip,jp'}$ = set of 2nd level nodes associated with stream connecting source i plant p to sink j plant p' using either a forward or backward branching scenario
$Z_{ip,jp'}$ = set of 3rd level nodes associated with stream connecting source i plant p to sink j plant p' using either a forward or backward branching scenario
$N_{ip,jp'}$ = Set of nth level nodes associated with stream connecting source i plant p to sink j plant p' using either a forward or backward branching scenario
X_{jp} = set of 1st level nodes associated with stream connecting freshwater mains to sink j plant p' using a forward branching scenario
Y_{jp} = set of 2nd level nodes associated with stream connecting freshwater mains to sink j plant p' using a forward branching scenario
Z_{jp} = set of 3rd level nodes associated with stream connecting freshwater mains to sink j plant p' using a forward branching scenario
N_{jp} = set of nth level nodes associated with stream connecting freshwater mains to sink j plant p' using a forward branching scenario
X_{ip} = set of 1st level nodes associated with stream connecting source i plant p to wastewater mains using a backward branching scenario
Y_{ip} = set of 2nd level nodes associated with stream connecting source i plant p to wastewater mains using a backward branching scenario
Z_{ip} = set of 3rd level nodes associated with stream connecting source i plant p to wastewater mains using a backward branching scenario
N_{ip} = set of nth level nodes associated with stream connecting source i plant p to wastewater mains using a backward branching scenario

Parameters

$z_{cjp}^{\min}$ = minimum permissible pollutant c composition in sink j, plant p, ppm
$z_{cjp}^{\max}$ = maximum permissible pollutant c composition in sink j, plant p, ppm
G_{jp} = flow rate required in sink j, plant p, kg/h
W_{ip} = flow rate available in source i, plant p, kg/h
$x_{c,ip}^{\text{Source}}$ = pollutant c composition in source i, plant p, ppm
x_c^{FRESH} = pollutant c composition in external freshwater, ppm
ρ = density, kg/m^3
μ = viscosity, kg/m s
α = coefficient associated with piping cost calculations
β = power coefficient associated with piping cost calculations
C^{FRESH} = cost of freshwater, \$/kg
H_y = operating hours per year, h/yr
γ = annualized piping cost factor, yr^{-1}
$L_{ipjp'}^{a}$ = length of pipe segment up to the 1st level node a, carrying water from source i, plant p to sink j plant p'
L_{ip}^{a} = length of pipe segment up to 1st level node a, carrying wastewater from source i, plant p to the waste mainstream
L_{jp}^{a} = length of pipe segment up to 1st level node a, carrying freshwater from mainstream to sink j, plant p
$L_{ipjp'}^{a,b}$ = length of pipe segment from 1st level node a to 2nd level node b, carrying water from source i, plant p to sink j plant p'
$L_{ip}^{a,b}$ = length of pipe segment from 1st level node a to 2nd level node b carrying wastewater from source i, plant p to the waste mainstream
$L_{jp}^{a,b}$ = length of pipe segment from 1st level node a to 2nd level node b carrying freshwater from mainstream to sink j, plant p
$L_{ipjp'}^{a,b,c}$ = length of pipe segment from 2nd level node b to 3rd level node c, carrying water from source i, plant p to sink j plant p' through 1st level node a
$L_{ip}^{a,b,c}$ = LENGTH of pipe segment from 2nd level node b to 3rd level node c, carrying wastewater from source i, plant p to the waste mainstream through 1st level node a

$L_{jp}^{a,b,c}$ = length of pipe segment from 2nd level node b to 3rd level node c, carrying freshwater from mainstream to sink j, plant p through 1st level node a

$L_{ipjp'}^{a,b,c,..,n-1,n}$ = length of pipe segment from $(n-1)$th level node $(n-1)$ to nth level node n, carrying water from source i, plant p to sink j plant p through nodes a, b, and c onward)

$L_{ip}^{a,b,c,..,n-1,n}$ = length of pipe segment from $(n-1)$th level node $(n-1)$ to nth level node n, carrying wastewater from source i, plant p to the waste mainstream through nodes a, b, and c onward)

$L_{jp}^{a,b,c,..,n-1,n}$ = length of pipe segment from node $(n-1)$th level node $(n-1)$ to nth level node n, carrying freshwater from mainstream to sink j, plant p through nodes a, b, and c onward)

Continuous variables

z_{cjp}^{in} = pollutant c composition in sink j, plant p

$M_{ipjp'}$ = mass flow rate from source i, plant p to sink j plant p'

F_{jp} = external freshwater mass flow rate required in sink j, plant p

D_{ip} = wastewater mass flow rate discharged by source i, plant p

$M_{ipjp'}^{a}$ = flow rate in pipe segment up to the 1st level node a, carrying water from source i, plant p to sink j plant p'

D_{ip}^{a} = flow rate in pipe segment up to 1st level node a, carrying wastewater from source i, plant p to the waste mainstream

F_{jp}^{a} = flow rate in pipe segment up to 1st level node a, carrying freshwater from mainstream to sink j, plant p

$M_{ipjp'}^{a,b}$ = flow rate in pipe segment from 1st level node a to 2nd level node b, carrying water from source i, plant p to sink j plant p'

$D_{ip}^{a,b}$ = flow rate in pipe segment from 1st level node a to 2nd level node b carrying wastewater from source i, plant p to the waste mainstream

$F_{jp}^{a,b}$ = flow rate in pipe segment from 1st level node a to 2nd level node b carrying freshwater from mainstream to sink j, plant p

$M_{ipjp'}^{a,b,c}$ = flow rate in pipe segment from 2nd level node b to 3rd level node c, carrying water from source i, plant p to sink j plant p' through 1st level node a

$D_{ip}^{a,b,c}$ = flow rate in pipe segment from 2nd level node b to 3rd level node c, carrying wastewater from source i, plant p to the waste mainstream through 1st level node a

$F_{jp}^{a,b,c}$ = Flow rate in pipe segment from 2nd level node b to 3rd level node c, carrying freshwater from mainstream to sink j, plant p through 1st level node a

$M_{ipjp'}^{a,b,c,...,n-1,n}$ = flow rate in pipe segment from $(n-1)$th level node $(n-1)$ to nth level node n, carrying water from source i, plant p to sink j plant p through nodes a, b, and c onward)

$D_{ip}^{a,b,c,..,n-1,n}$ = flow rate in pipe segment from $(n-1)$th level node $(n-1)$ to nth level node n, carrying wastewater from source i, plant p to the waste mainstream through nodes a, b, and c onward)

$F_{jp}^{a,b,c,..,n-1,n}$ = flow rate in pipe segment from node $(n-1)$th level node $(n-1)$ to nth level node n, carrying freshwater from mainstream to sink j, plant p through nodes a, b, and c onward)

$DI_{ipjp'}^{a}$ = diameter of pipe segment up to the 1st level node a, carrying water from source i, plant p to sink j plant p'

DI_{ip}^{a} = diameter of pipe segment up to 1st level node a, carrying wastewater from source i, plant p to the waste mainstream

DI_{jp}^{a} = diameter of pipe segment up to 1st level node a, carrying freshwater from mainstream to sink j, plant p

$DI_{ipjp'}^{a,b}$ = diameter of pipe segment from 1st level node a to 2nd level node b, carrying water from source i, plant p to sink j plant p'

$DI_{ip}^{a,b}$ = diameter of pipe segment from 1st level node a to 2nd level node b carrying wastewater from source i, plant p to the waste mainstream

$DI_{jp}^{a,b}$ = diameter of pipe segment from 1st level node a to 2nd level node b carrying freshwater from mainstream to sink j, plant p

$DI_{ipjp'}^{a,b,c}$ = diameter of pipe segment from 2nd level node b to 3rd level node c, carrying water from source i, plant p to sink j plant p' through 1st level node a

$DI_{ip}^{a,b,c}$ = diameter of pipe segment from 2nd level node b to 3rd level node c, carrying wastewater from source i, plant p to the waste mainstream through 1st level node a

$DI_{jp}^{a,b,c}$ = diameter of pipe segment from 2nd level node b to 3rd level node c, carrying freshwater from mainstream to sink j, plant p through 1st level node a

$DI_{ipjp'}^{a,b,c,...,n-1,n}$ = diameter of pipe segment from $(n-1)$th level node $(n-1)$ to nth level node n, carrying water from source i, plant p to sink j plant p through nodes a, b, and c onward)

$DI_{ip}^{a,b,c,...,n-1,n}$ = diameter of pipe segment from $(n-1)$th level node $(n-1)$ to nth level node n, carrying wastewater from source i, plant p to the waste mainstream through nodes a, b, and c onward)

$DI_{jp}^{a,b,c,...,n-1,n}$ = diameter of pipe segment from node $(n-1)$th level node $(n-1)$ to nth level node n, carrying freshwater from mainstream to sink j, plant p through nodes a, b, and c onward)

Literature Cited

1. Wang YP, Smith R. Wastewater minimisation. *Chem Eng Sci*. 1994; 49(7):981–1006.
2. Wang YP, Smith R. Design of distributed effluent treatment systems. *Chem Eng Sci*. 1994;49(18):3127–3145.
3. Olesen SG, Polley GT. Dealing with plant geography and piping constraints in water network design. *Process Saf Environ Prot*. 1996; 74(4):273–276.
4. Alva-Argáez A, Kokossis AC, Smith R. Wastewater minimisation of industrial systems using an integrated approach. *Comput Chem Eng*.1998;22 Supplement 1(0):S741–S744.
5. Savelski MJ, Bagajewicz MJ. On the optimality conditions of water utilization systems in process plants with single contaminants. *Chem Eng Sci*. 2000;55(21):5035–5048.
6. El-Halwagi MM, Gabriel F, Harell D. Rigorous graphical targeting for resource conservation via material recycle/reuse networks. *Ind Eng Chem Res*. 2003;42(19):4319–4328.
7. Manan ZA, Tan YL, Foo DCY. Targeting the minimum water flow rate using water cascade analysis technique. *AIChE J*. 2004;50(12): 3169–3183.
8. Prakash R, Shenoy UV. Targeting and design of water networks for fixed flowrate and fixed contaminant load operations. *Chem Eng Sci*. 2005;60(1):255–268.
9. Liu Z-Y, Yang Y-Z, Zhang Y. Determining the pinch point and calculating the freshwater target for water-using systems with single contaminant. *Chem Eng Res Des*. 2007;85(11):1485–1490.
10. Hu N, Feng X, Deng C. Optimal design of multiple-contaminant regeneration reuse water networks with process decomposition. *Chem Eng J*. 2011;173(1):80–91.
11. Lee J-Y, Chen C-L, Wen T-L, Ng DKS, Foo DCY, Wang T-C. Synthesis and design of chilled water networks using mathematical optimization. *Appl Therm Eng*. 2013;58(1–2):638–649.
12. Chaturvedi ND, Bandyopadhyay S. Simultaneously targeting for the minimum water requirement and the maximum production in a batch process. *J Clean Prod*. DOI: 10.1016/j.jclepro.2013.11.079.
13. Liao ZW, Wu JT, Jiang BB, Wang JD, Yang YR. Design methodology for flexible multiple plant water networks. *Ind Eng Chem Res*. 2007;46(14):4954–4963.
14. Lovelady EM, El-Halwagi MM. Design and integration of eco-industrial parks for managing water resources. *Environ Prog Sustain Energy*. 2009;28(2):265–272.
15. Chen C-L, Hung S-W, Lee J-Y. Design of inter-plant water network with central and decentralized water mains. *Comput Chem Eng*. 2010;34(9):1522–1531.
16. Aviso K, Tan R, Culaba A. Designing eco-industrial water exchange networks using fuzzy mathematical programming. *Clean Technol Environ Policy*. 2010;12(4):353–363.
17. Chew IML, Foo DCY, Ng DKS, Tan RR. Flowrate targeting algorithm for interplant resource conservation network. Part 1: unassisted integration scheme. *Ind Eng Chem Res*. 2010;49(14):6439–6455.
18. Rubio-Castro E, Ponce-Ortega JM, Serna-González M, Jiménez-Gutiérrez A, El-Halwagi MM. A global optimal formulation for the

water integration in eco-industrial parks considering multiple pollutants. *Comput Chem Eng*. 2011;35(8):1558–1574.
19. Rubio-Castro E, Ponce-Ortega JM, Serna-González M, El-Halwagi MM. Optimal reconfiguration of multi-plant water networks into an eco-industrial park. *Comput Chem Eng*. 2012;44(0):58–83.
20. Boix M, Montastruc L, Pibouleau L, Azzaro-Pantel C, Domenech S. Industrial water management by multiobjective optimization: from individual to collective solution through eco-industrial parks. *J Clean Prod*. 2012;22(1):85–97.
21. Lee J-Y, Chen C-L, Lin C-Y, Foo DCY. A two-stage approach for the synthesis of inter-plant water networks involving continuous and batch units. *Chem Eng Res Des*. 2014;92(5):941–953.
22. Alnouri S, Linke P, El-Halwagi MM. Water Integration in Industrial Zones—a Spatial Representation with Direct Recycle Applications. *Clean Technol Environ Policy*. DOI: 10.1007/s10098-014-0739-2.
23. Bishnu SK, Linke P, Alnouri SY, El-Halwagi MM. Multiperiod planning of optimal industrial city direct water reuse networks. *Ind Eng Chem Res*. 2014;53(21):8844–8865.
24. Wong P, Larson R. Optimization of natural gas pipeline system via dynamic programming. *IEEE Trans Automat Control*. 1968;9:5–13.
25. Graham GE, Maxwell DA, Vallane A. How to optimize gas pipeline networks. *Pipelines Ind*. 1971:41–43.
26. Flanigan O. Constrained derivatives in natural gas pipeline system optimization. *J Pet Technol*. 1972:549–556.
27. Baskaran S, Salzborn FJ. Optimal design of gas pipeline networks. *J Oper Res Soc*. 1979;30:1047–1060.
28. Olorunniwo FO, Jensen PA. Dynamic sizing and locationing of facilities on natural gas transmission networks. *Eng Optim*. 1982;6: 95–109.
29. Olorunniwo FO, Jensen PA. Optimal capacity expansion policy for natural gas transmission network—a decomposition approach. *Eng Optim*. 1982;6:13–30.
30. Almisned O, Alkahtani R. An optimal pipeline network for transporting natural gas among the gulf countries. In: *The Second Symposium on Technologies, Economics, and Investment Opportunities in Petrochemical Industries*. Kingdom of Saudi Arabia,1996.
31. Amado A, José R. *A Multi-Commodity Network Flow Approach for Sequencing Refined Products in Pipeline Systems*, PhD diss., University of Tennessee, 2011. http://trace.tennessee.edu/utk_graddiss/940.
32. Bonnans JF, Spiers G, Vie J-L. *Global Optimization of Pipe Networks by the Interval Analysis Approach: The Belgium Network Case*. Rapport de recherche (2011) 13 [hal-00642932 - version 1], 2011.
33. Peters MS, Timmerhaus KD, West RE. *Plant Design and Economics for Chemical Engineers (International Edition)*. New York: McGraw-Hill Higher Education, 2003.
34. Lindo Systems, What'sBest! 9.0 - Excel Add-In for Linear, Nonlinear, and Integer Modeling and Optimization. Available at: www.lindo.com/index.php?option=com_content&view=article&id=3&Itemid=11; accessed on 26 February, 2014.

Manuscript received Apr. 22, 2014, and revision received May 27, 2014.